AI & Robust Optimization for Social Good

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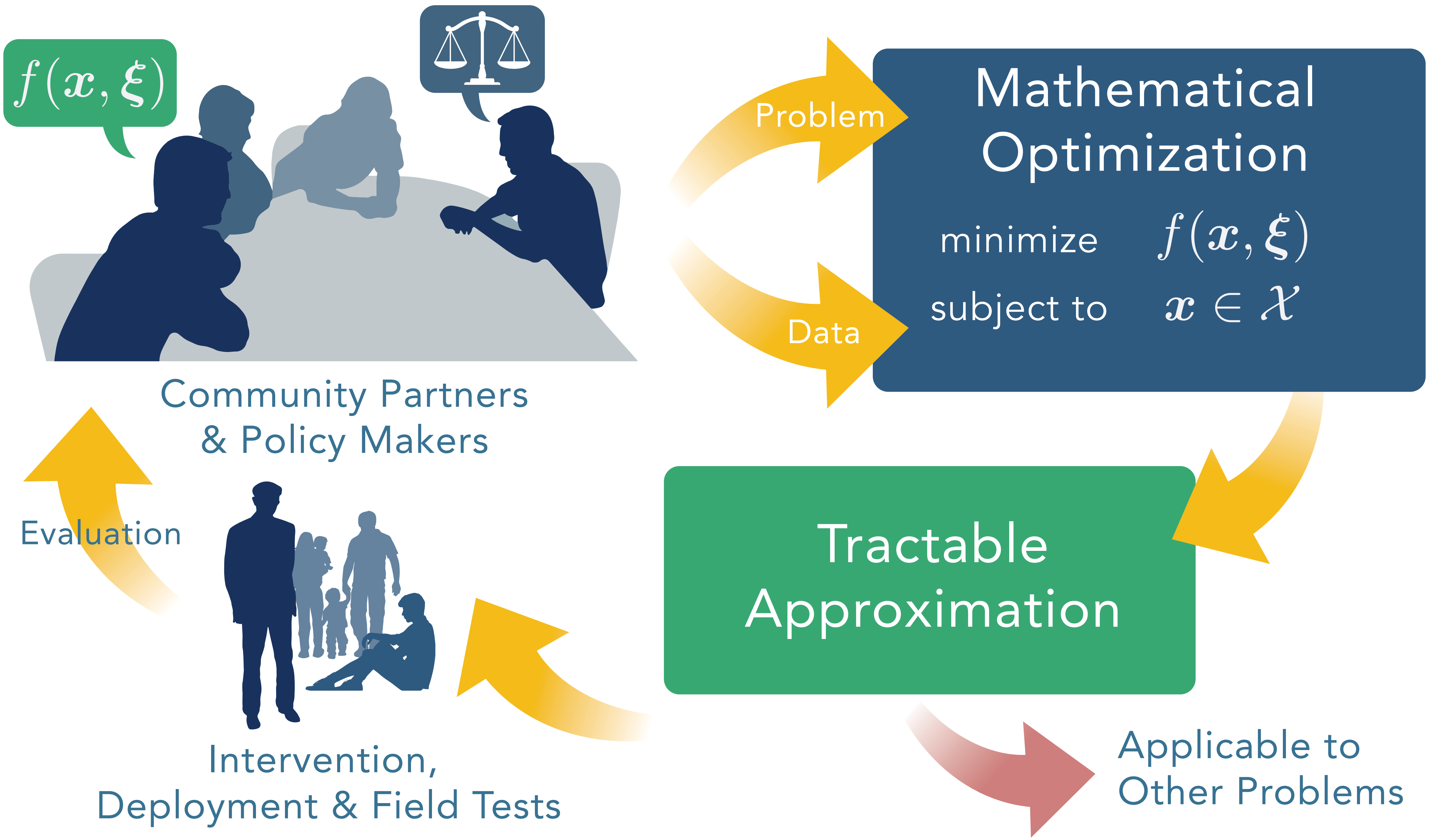
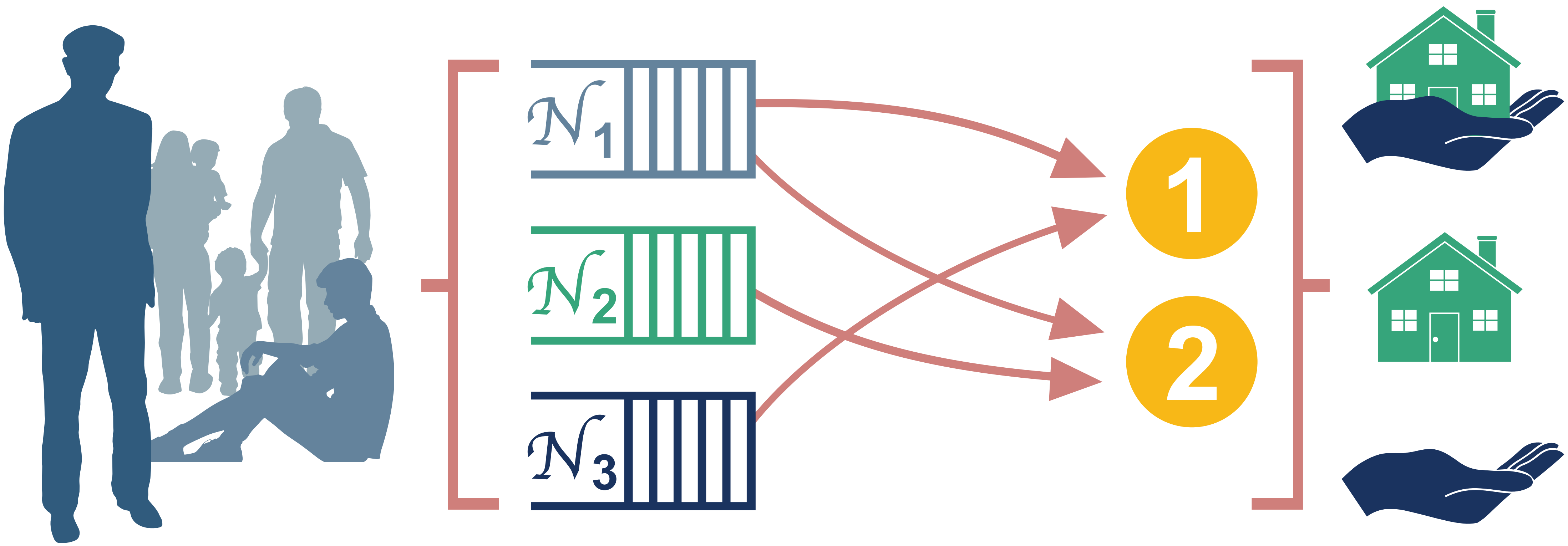
Assistant Professor, ISE and CS
University of Southern California

End-to-End Research Approach



2 Types of Resource Constrained Interventions!

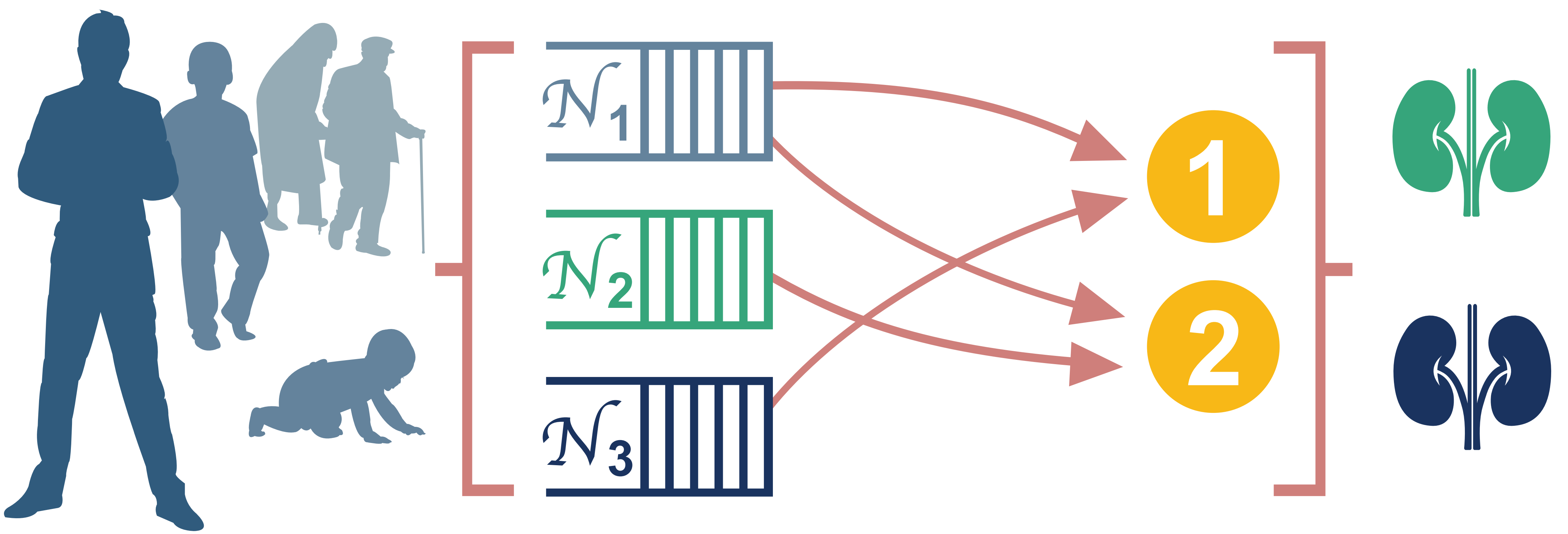
Resource Constrained social Interventions



Queuing System / Waitlist: e.g., public housing, kidneys for transplantation, costly treatments, etc.

Resource Constrained Interventions

social



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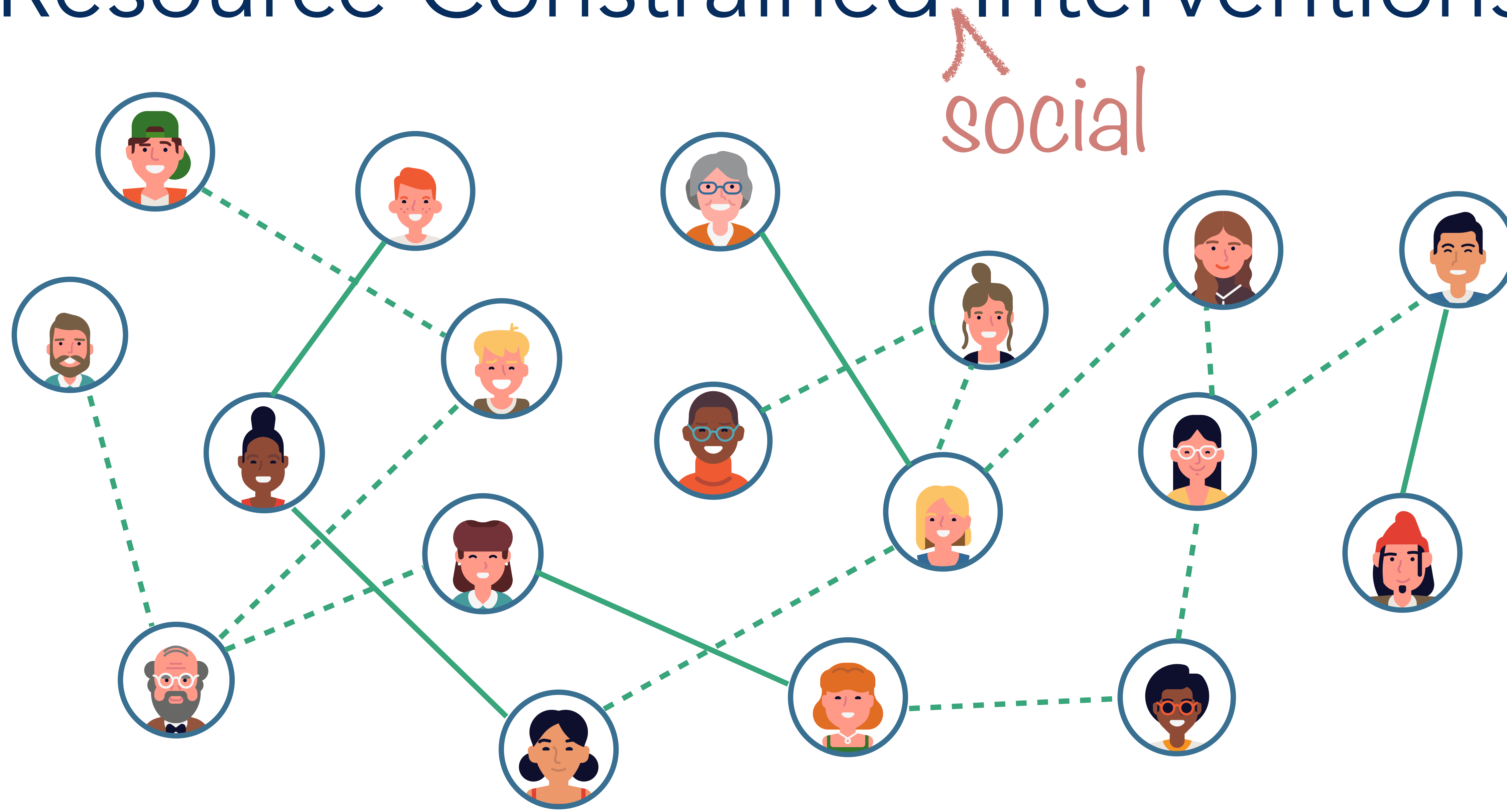

Social Network Based Interventions: e.g., suicide prevention, substance abuse prevention, etc.

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Resource Constrained Interventions

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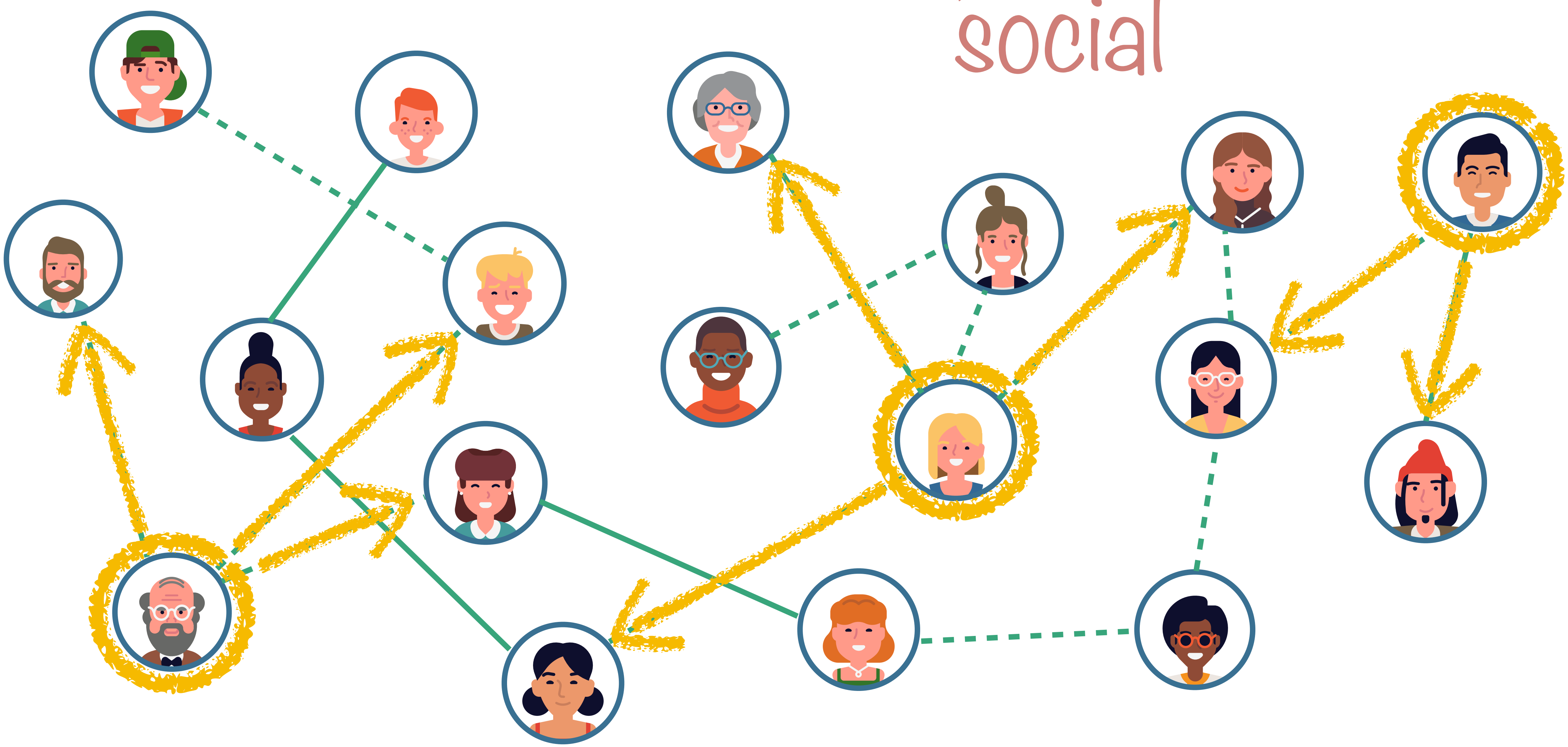

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Common Threads

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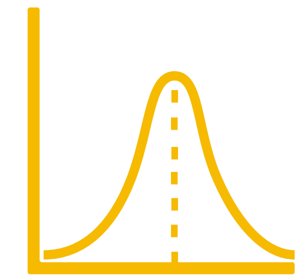
Uncertainty



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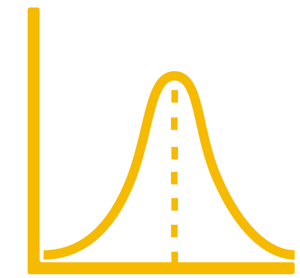
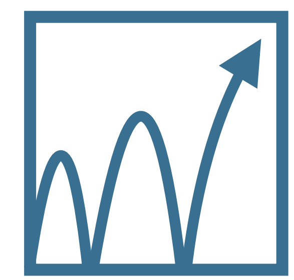


Dynamics



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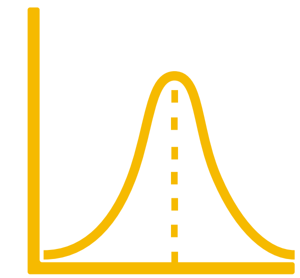

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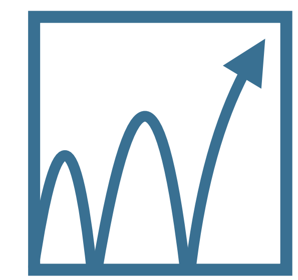
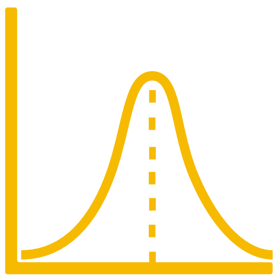
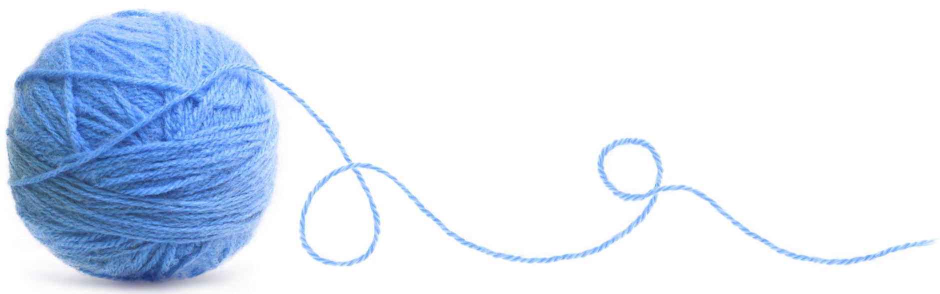


Scalability

Common Threads

Uncertainty

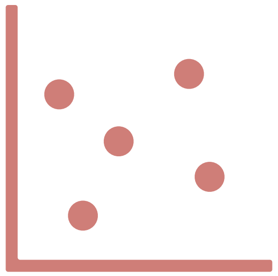




Dynamics

Scalability

Data



Permeating Themes

Heterogeneous Population

Heterogenous Resources

Socially Sensitive Settings

Permeating Themes

Heterogeneous Population

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Heterogeneous Population

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Socially Sensitive Settings

Fairness & Personalization

Outline

- ☐ Estimating Wait Times in Resource Allocation Systems
- ☐ Designing Policies for Allocating Scarce Resources
    - ☐ Preference Elicitation
    - ☐ Policy Optimization
- ☐ Optimizing "Gatekeeper Trainings" for Suicide Prevention

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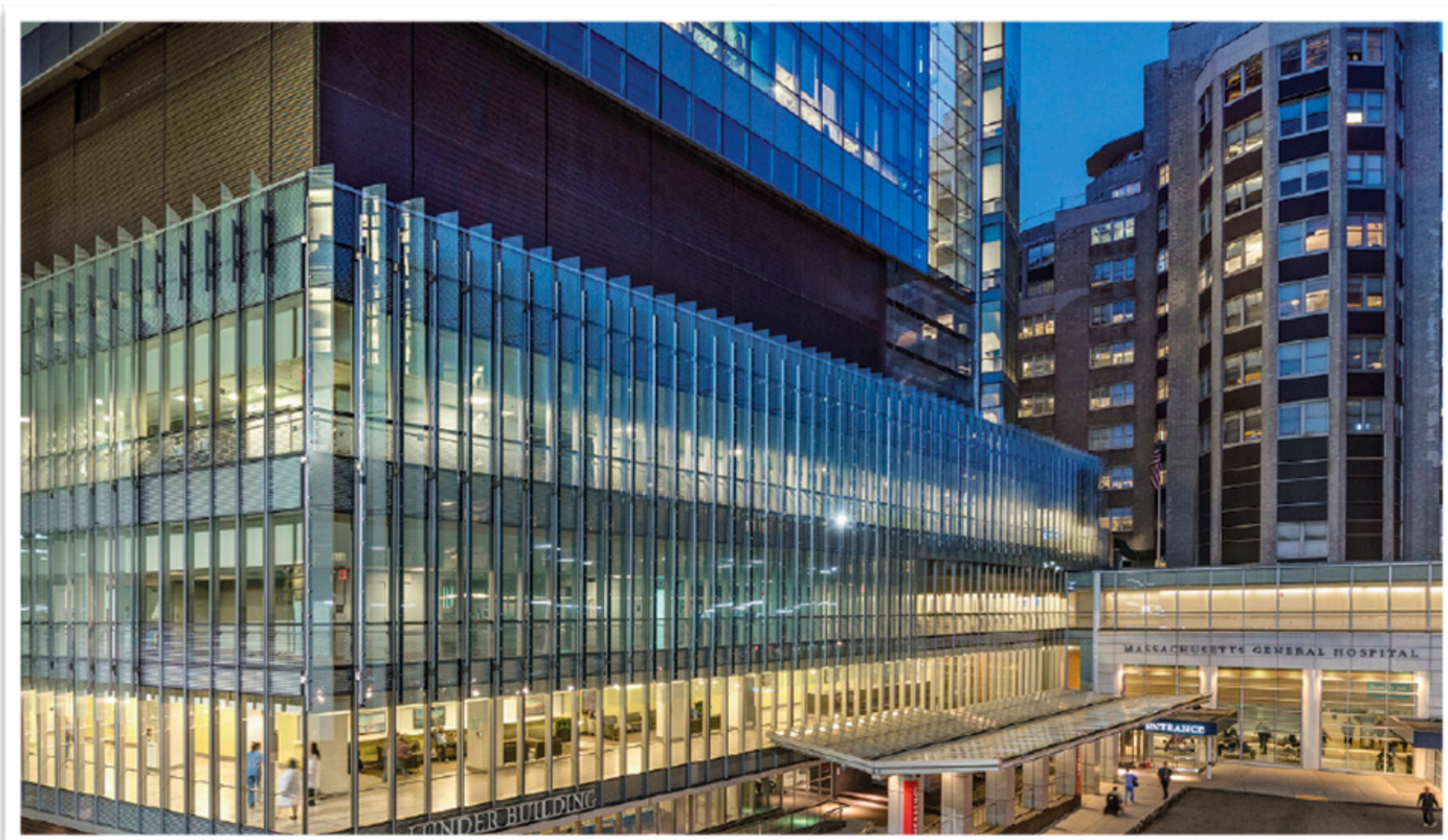
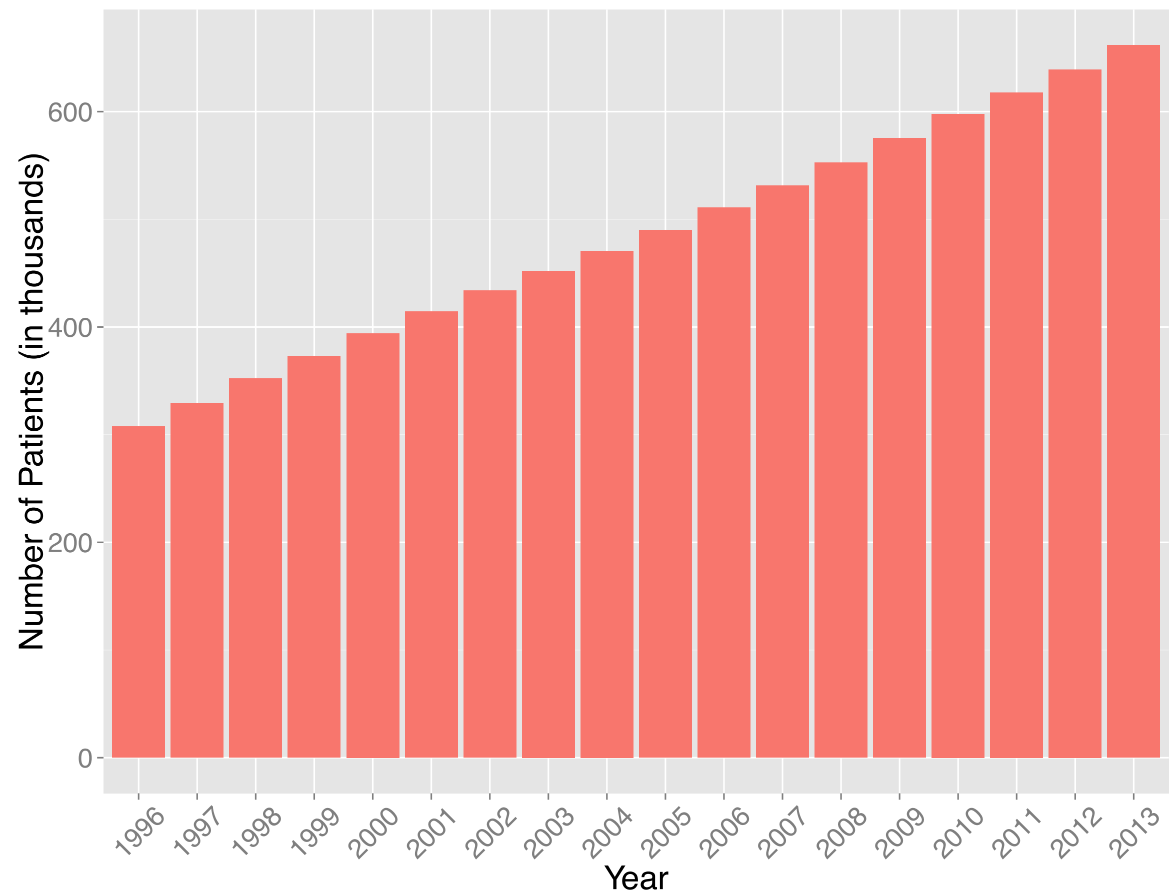
Partner



MASSACHUSETTS
GENERAL HOSPITAL

End-Stage Renal Disease

source: https://www.usrds.org



- Terminal disease affecting >600,000 patients in U.S.
- Dialysis vs. kidney transplant (preferred)
- Living donors vs. deceased donors

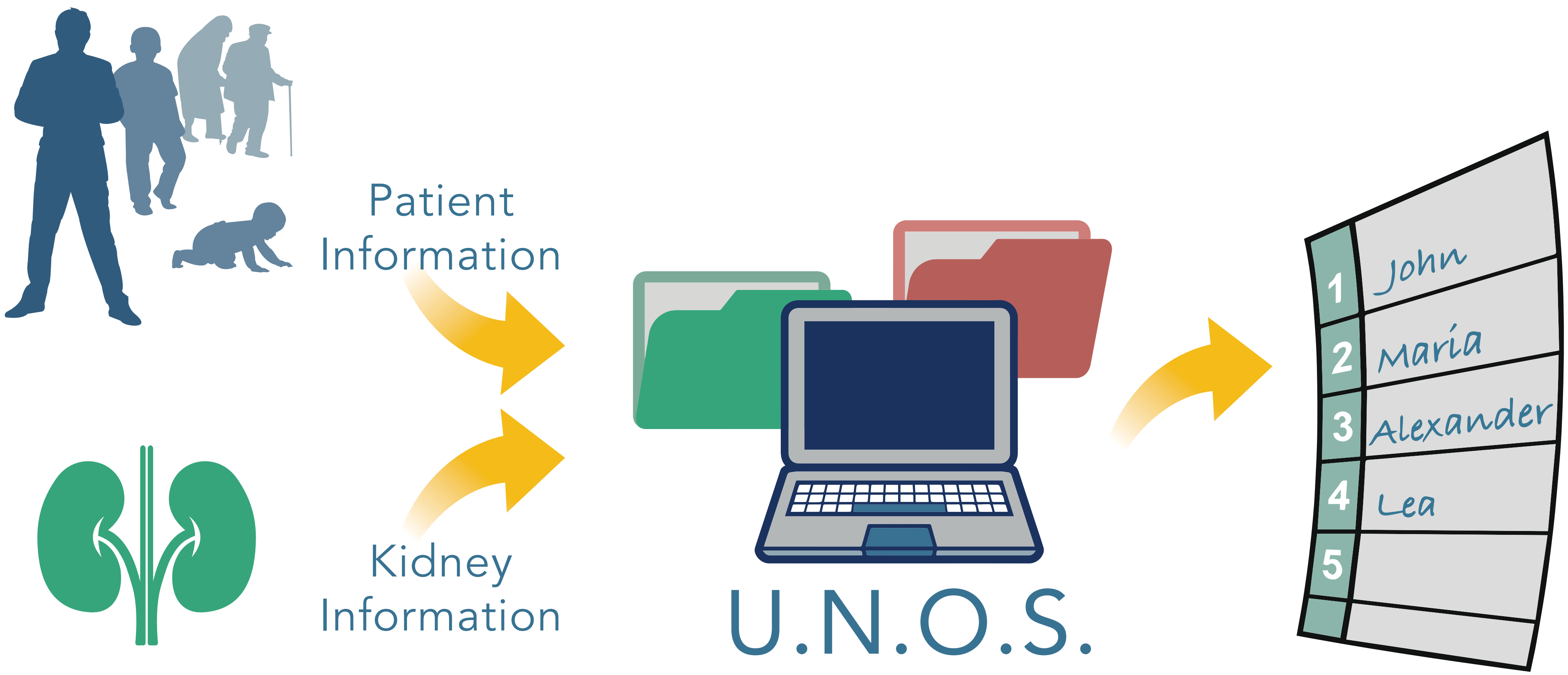
Organ Shortage

- 100k patients waiting
- 36k additions per year
- 19k transplants/year
  - 13.4k (70%) from deceased donors
  - 5.6k (30%) from living donors

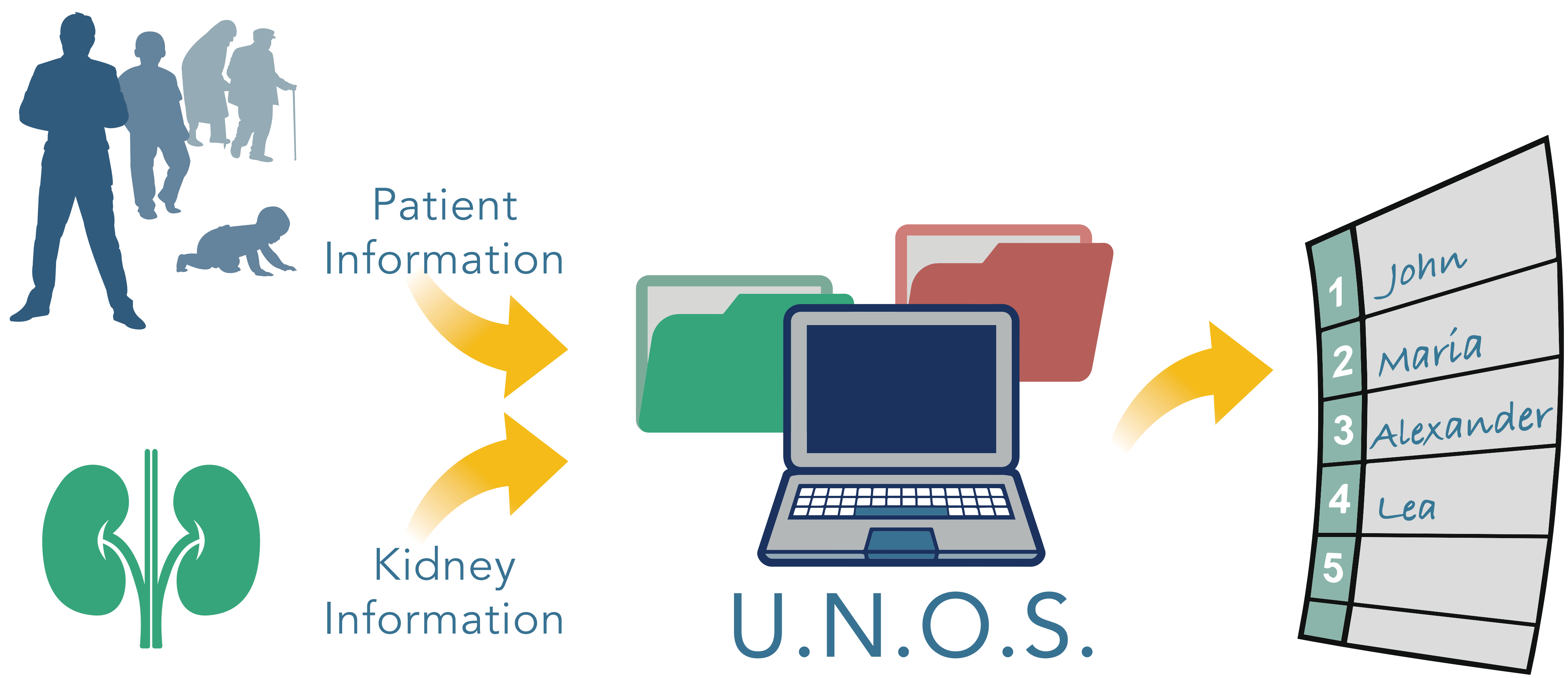
Organ Shortage

	3-yr trend
- 100k patients waiting	+20%
- 36k additions per year	
- 19k transplants/year	
- 13.4k (70%) from deceased donors	+20%
- 5.6k (30%) from living donors	-2%

U.S. Kidney Allocation System



U.S. Kidney Allocation System



- Medical compatibility: blood group, weight, etc.
- Geographic proximity (24-36 hours to transplant)
- *Point based:* wait time, blood antigens: **~FCFS**

Wait Time Estimation

Personalized Estimates:

Patient X of blood type O is listed in a given geographic region. He is currently ranked 50th. How long until he receives an offer of a particular quality?

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Patient X of blood type O is listed in a given geographic region. He is currently ranked 50th. How long until he receives an offer of a particular quality?

- Important for:
  - ☑ dialysis management
  - ☑ planning of daily life activities
  - ☑ accept/reject decisions

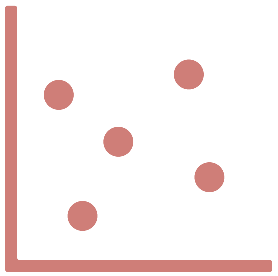
Wait Time Estimation

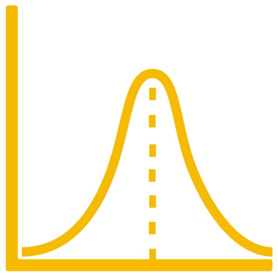
Personalized Estimates:

Patient X of blood type O is listed in a given geographic region. He is currently ranked 50th. How long until he receives an offer of a particular quality?

Interpretability: Answer in the Form of Quantiles!

Challenges





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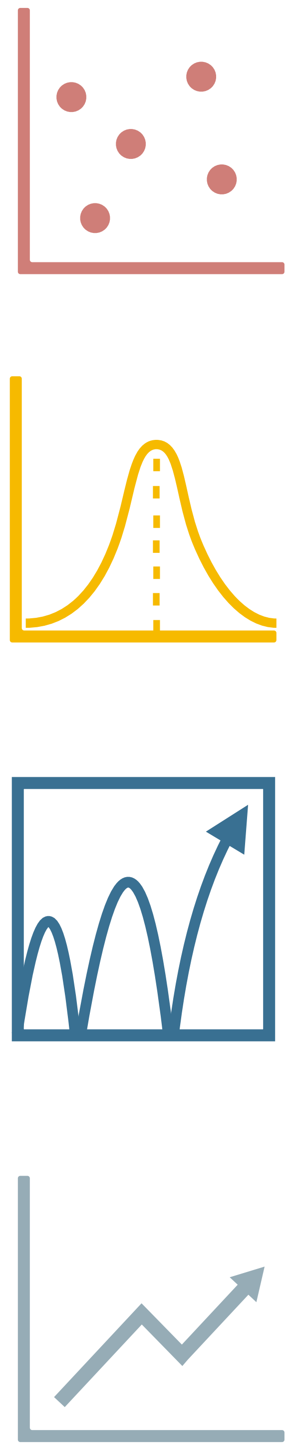
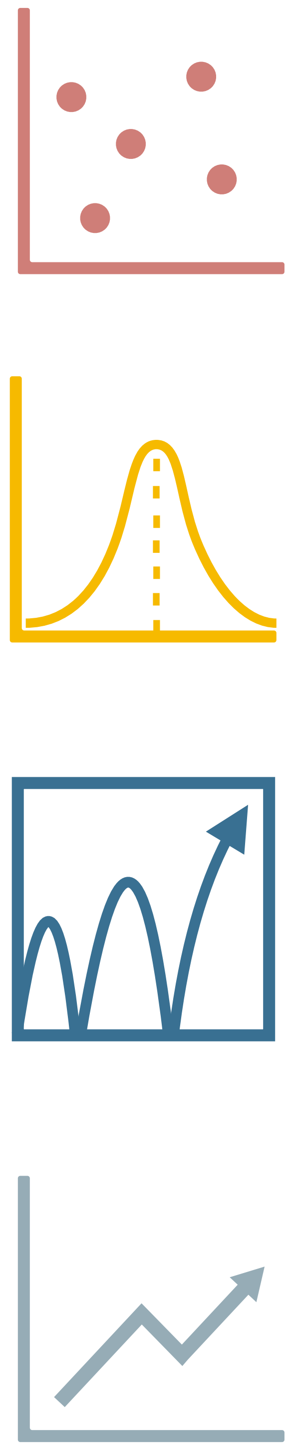


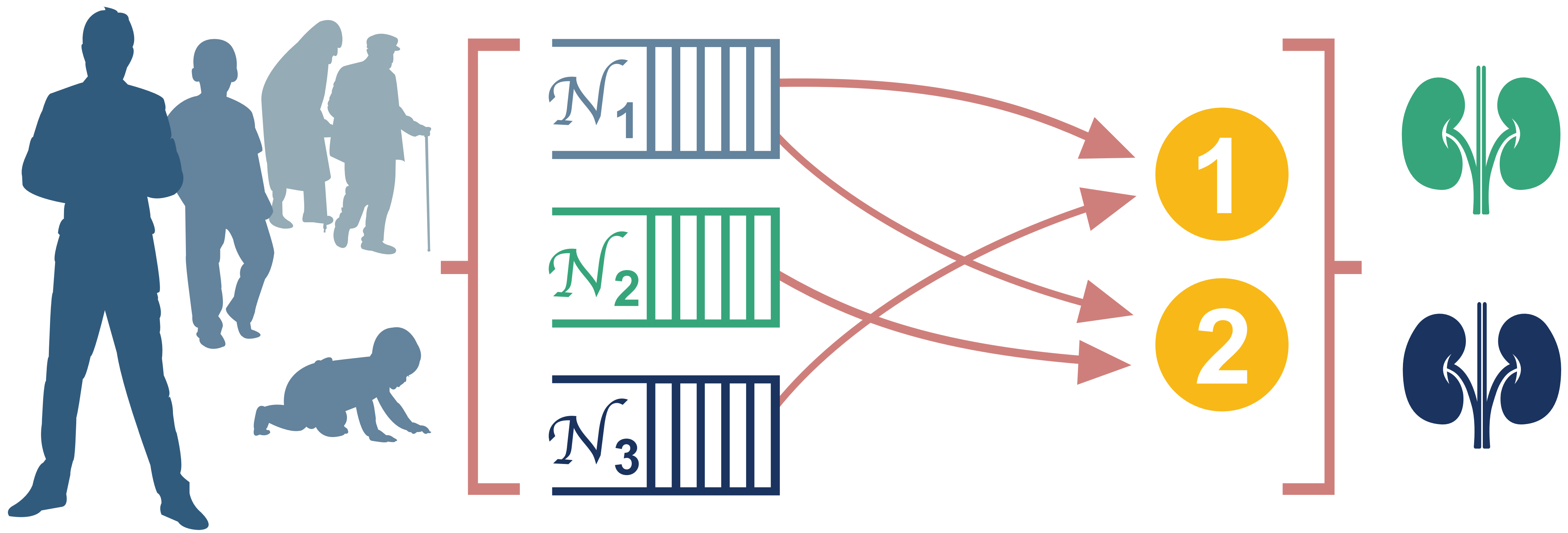
- Predicting accept/decline decisions is already hard:
  - Kim *et al* 15: use all available historical data, build series of prediction models (log. reg., SVM, CART, RF); error rates vary 22-47%

Challenges



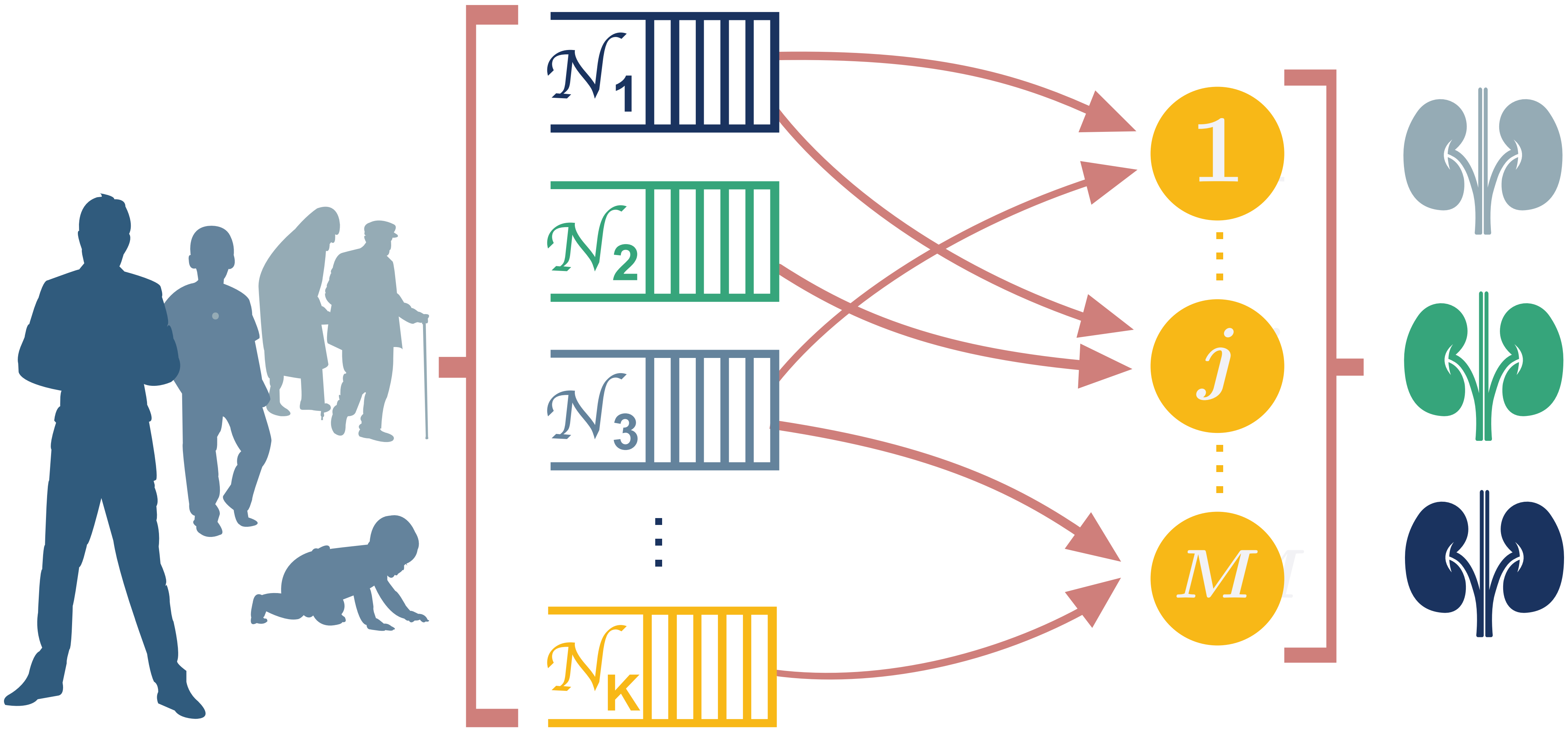
- Predicting accept/decline decisions is already hard:
  - Kim *et al* 15: use all available historical data, build series of prediction models (log. reg., SVM, CART, RF); error rates vary 22-47%
- In practice:
  - Incomplete information: other patients' preferences
  - Unstable/ non-stationary system

Multiclass Multiserver Queuing

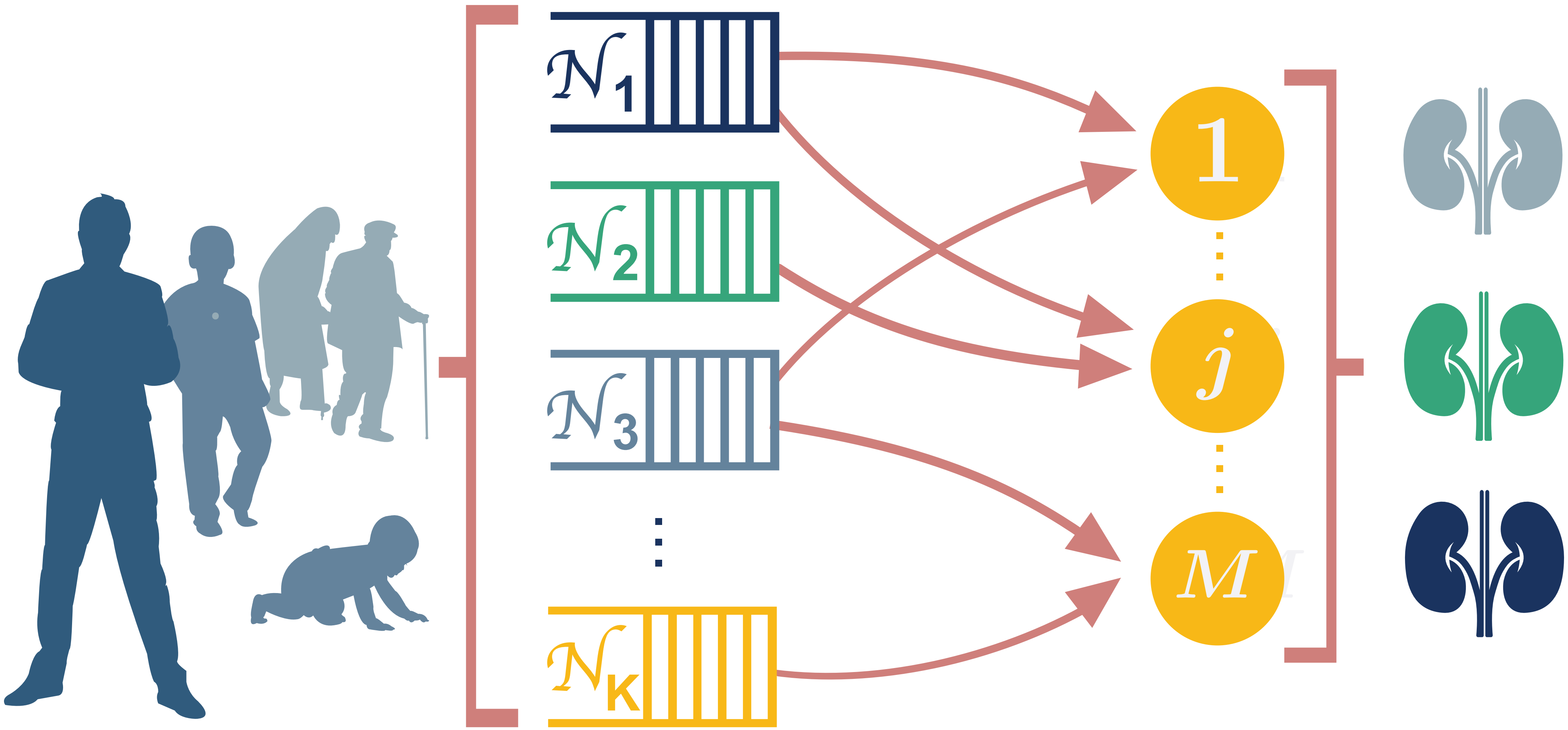


- Multiclass, multiserver (MCMS) queuing system
  - Servers: resource types
  - Customer classes/queues: preferences

MCMS under FCFS

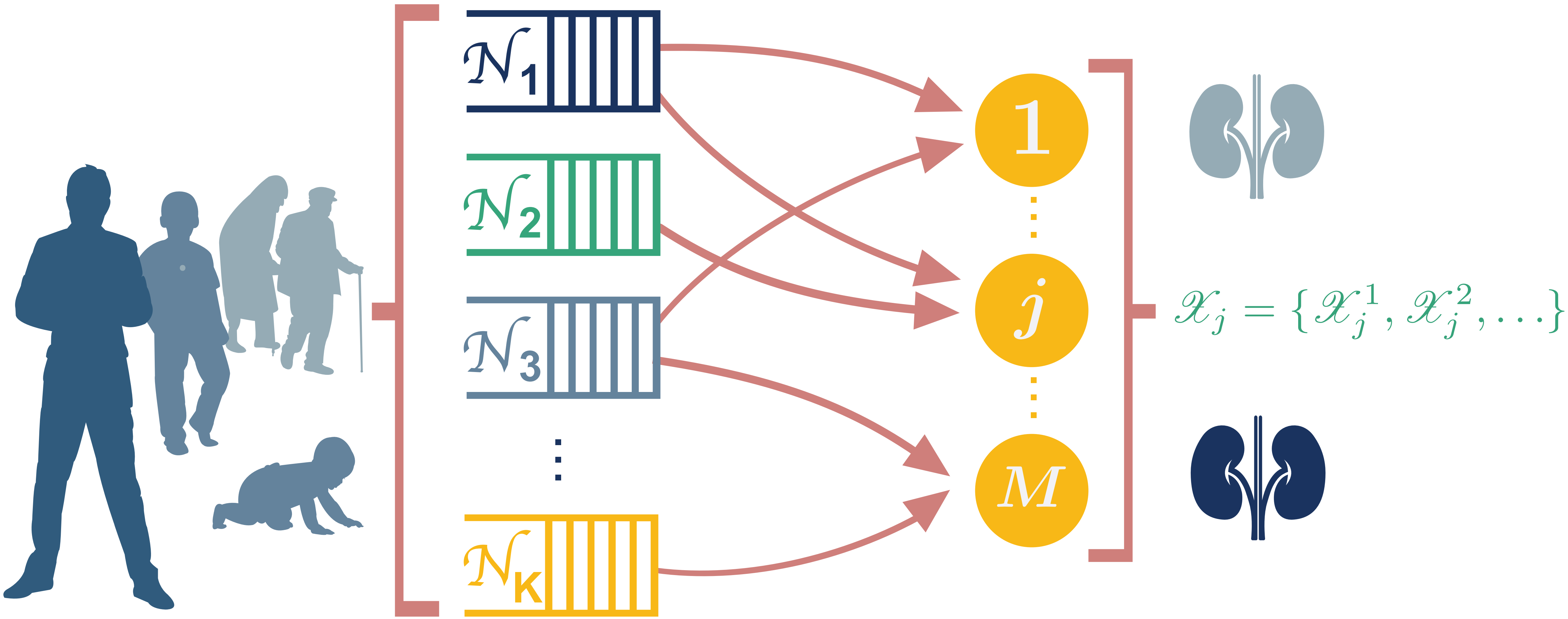


MCMS under FCFS



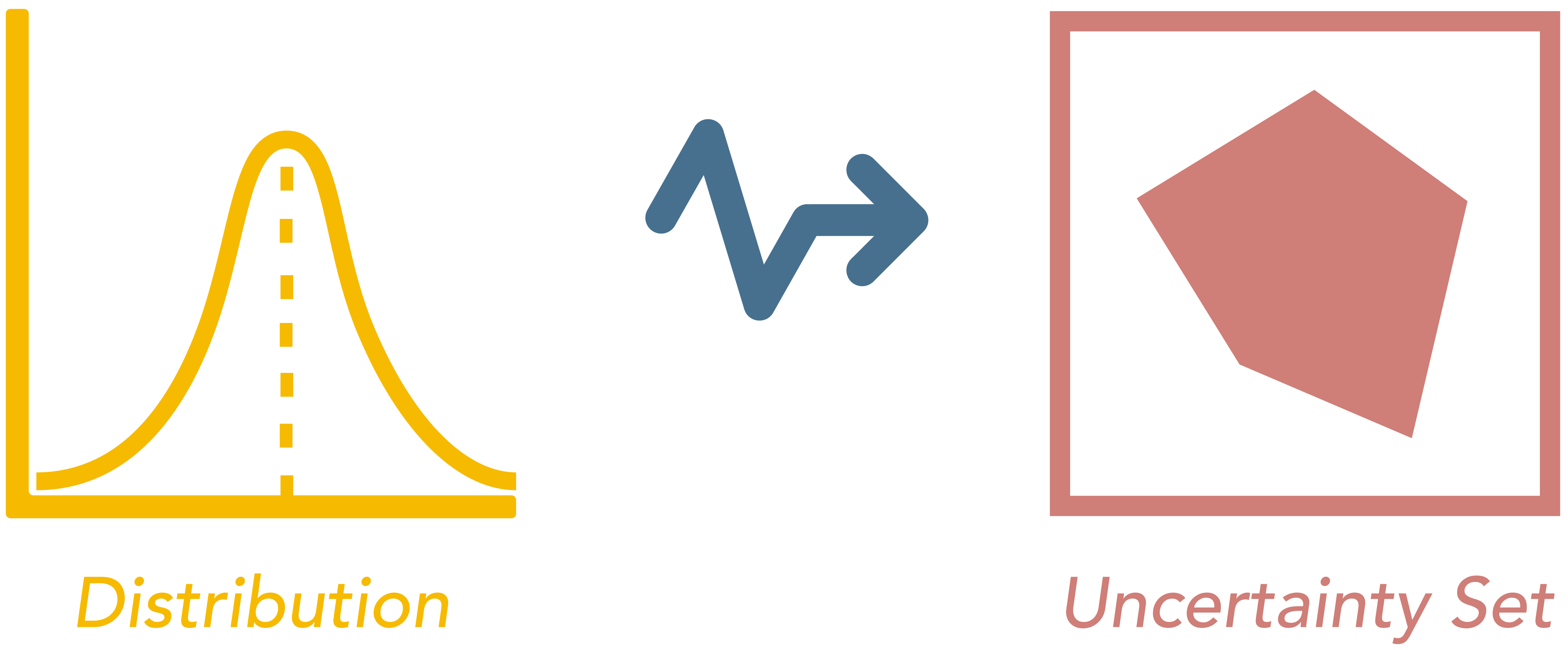
- $\sigma(\nu)$ arrival order of customer $\nu \in \{1, \ldots, \sum_i \mathcal{N}_i\}$

MCMS under FCFS

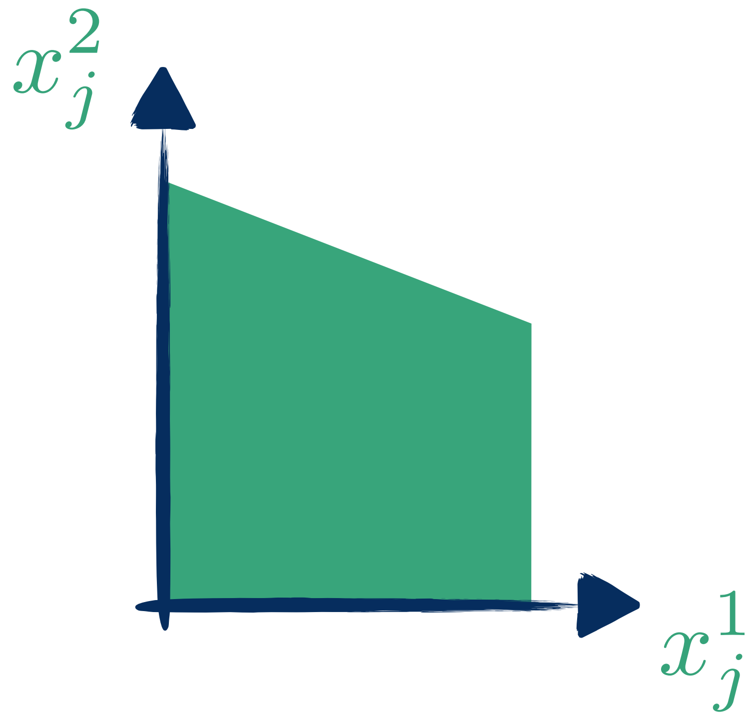


- $\sigma(\nu)$ arrival order of customer $\nu \in \{1, \dots, \sum_i \mathscr{N}_i\}$
- $\mathscr{W}_i(\mathscr{N}_1, \dots, \mathscr{N}_K, \sigma, \mathscr{X}_1, \dots, \mathscr{X}_M)$ clearing time of queue i

Robust Optimization



Model of Uncertainty



- Service times:

$$\mathbb{X}_j = \left\{ x_j \in \mathbb{R}^{\bar{\ell}_j} \; : \; \sum_{k=1}^{\ell} x_j^k \leq \frac{\ell}{\mu_j} + \Gamma_j^{\mathbb{X}} (\ell)^{1/\alpha_j}, \; \ell = 1, \ldots, \bar{\ell}_j \right\}$$

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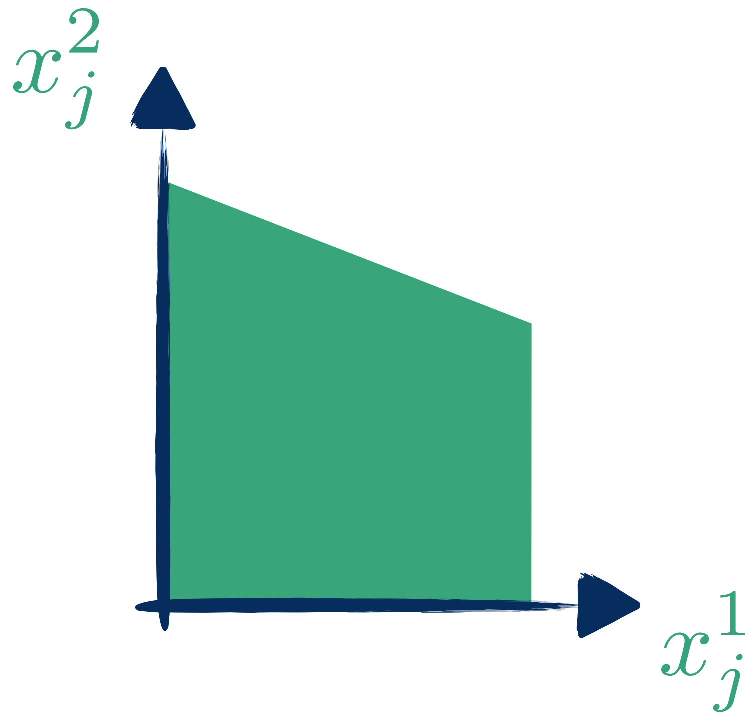

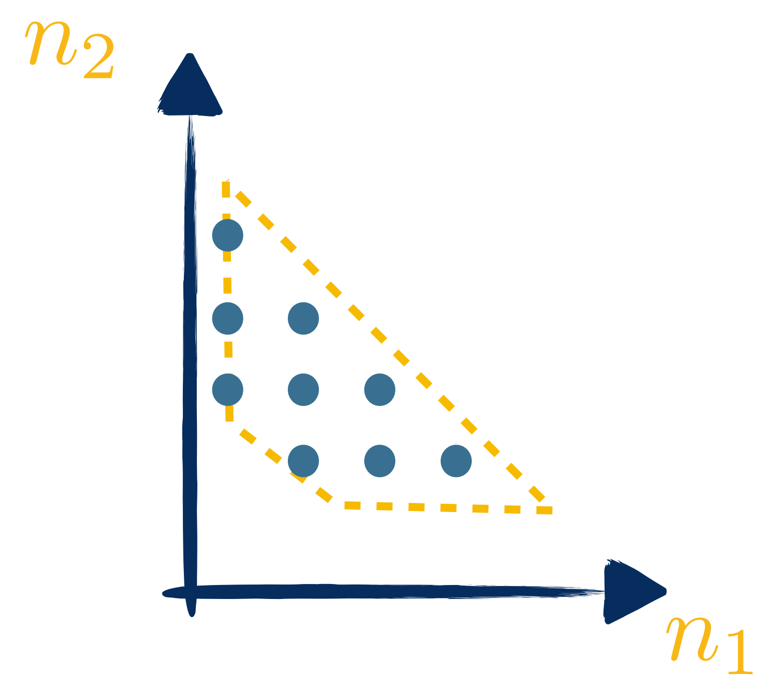

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- Population vector: $n \in \mathbb{P} \cap \mathbb{N}^K$

- Arrival order: $\sigma \in \Sigma(n)$

Robust Wait Times

$$
\begin{aligned}
W_i: \quad & \text{maximize} \quad && \mathscr{W}_i(n_1, \ldots, n_K, \sigma, x_1, \ldots, x_M) \\
& \text{subject to} \quad && n \in \mathbb{P} \cap \mathbb{N}^K \\
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& && x_j \in \mathbb{X}_j, \quad j = 1, \ldots, M
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NP-Hard!

Robust Wait Times

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NP-Hard!

- No tractable expression for $\mathscr{W}_i$
  - Lindley equations break down
- Key idea: model assignment of servers to customers
  - y_{kj}^{ℓ}: ℓth service from server j assigned to class k

Robust Wait Times

Assignment-style formulation:

$$\begin{aligned} \text{maximize} \quad & w_i \\ \text{subject to} \quad & \sum_k y_{kj}^{\ell} \le 1, \quad \sum_{\ell,j} y_{kj}^{\ell} \le n_k \\ & \sum_{k'} y_{k'j}^{\ell} \ge f_{kj}^{\ell} \\ & w_k \le c_j^{\ell} + \bar{\zeta} f_{kj}^{\ell} \\ & w_k \ge c_j^{\ell} - \bar{\zeta}\left(1 - y_{kj}^{\ell}\right) \\ & (c, n) \in \text{uncertainty sets}, \ (y, f) \text{ binary} \end{aligned}$$

Performance: Accuracy

Estimation error vs simulation:

statistics	avg.	95-%ile	97-%ile	99-%ile
avg. abs. rel. error	6.52%	2.64%	2.55%	3.41%

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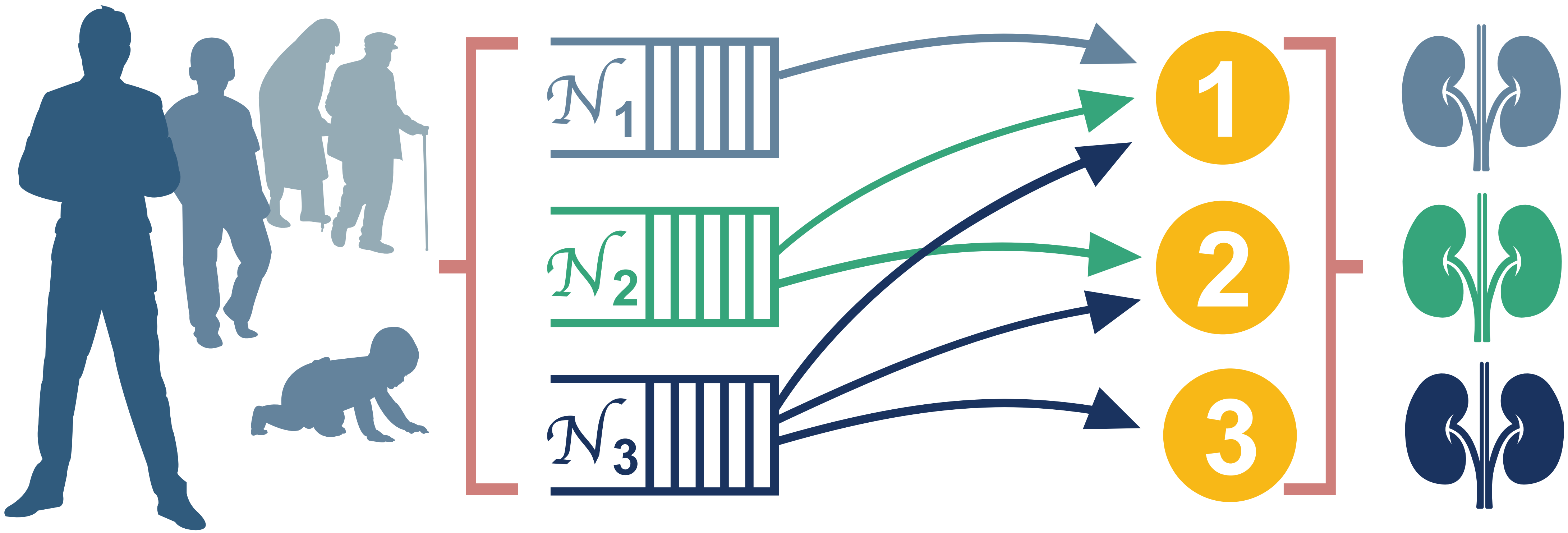
Estimation error when true distribution $\neq$ *assumed:*

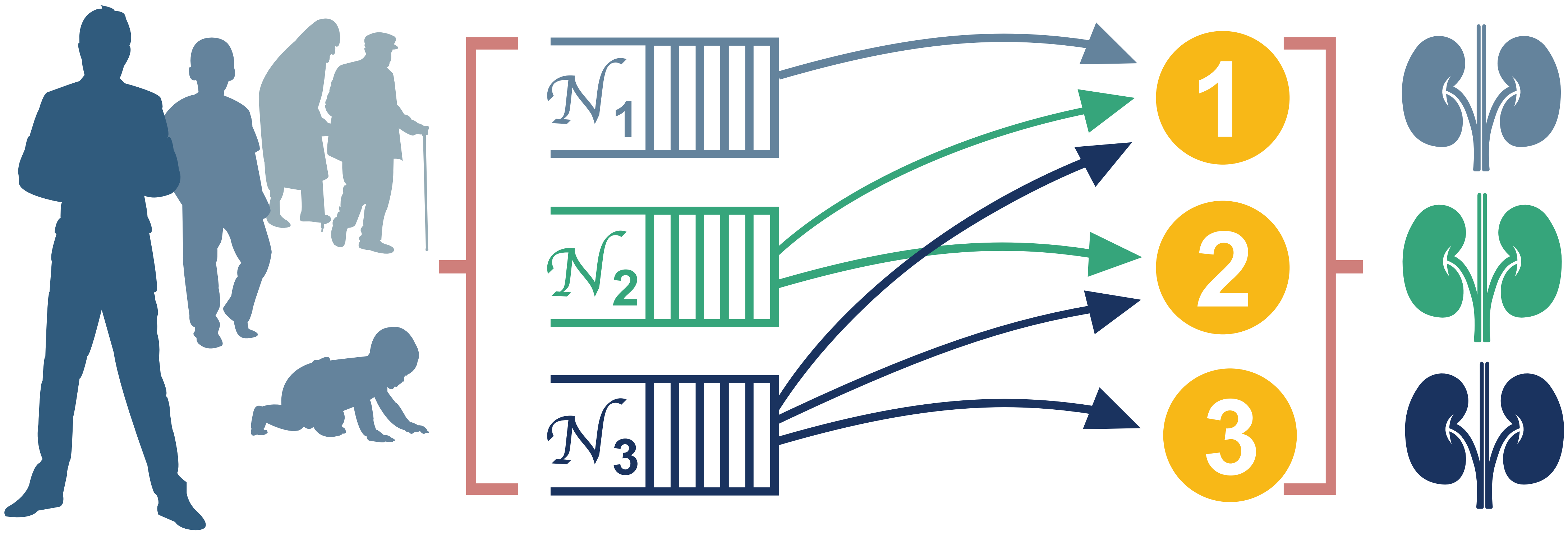
avg. queue population	5	100	500
simulation avg. abs. rel.	21%	15%	12%
our avg. abs. rel. error	13%	9%	7.5%

Hierarchical MCMS



- Hierarchy across resource types
- Server j provides jth ranked service
- Induces "threshold-type" customer preferences

Hierarchical MCMS



- Nested structure enables to strengthen formulations
- Robust wait time for service of any rank W_K
- Problem remains NP-hard

Scalable Heuristic

- View so far: individual assignments y_{kj}^{ℓ}
  - Scales with n
- Alternative view:
  - Aggregate assignments m_j
  - Independent of n

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$\widehat{W}_K$

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\begin{aligned}
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& \sum_{k=j}^{K} m_k \le \sum_{k=j}^{K} n_k + K - j \\
& n \in \mathbb{P}
\end{aligned}
$$

Scalable Heuristic

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SOCP!

Approximation Guarantee

- W_K exact robust wait time
- $\widehat{W}_K$ approximation

Let

$$\chi = \max_j \left\{ \frac{1}{\mu_j} + \Gamma_j^{\mathbb{X}} \right\}$$

For a hierarchical MCMS system,

$$W_K \leq \widehat{W}_K \leq W_K + 2\chi$$

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For a hierarchical MCMS system,

$$W_K \leq \widehat{W}_K \leq W_K + 2\chi$$

⇨ *approximation becomes tighter as n increases*

Heuristic: Performance

Computation Times for Different HMCMS Instances

	general MIP	SOCP
100 customers	1 sec	0.8 sec
1,000 customers	< 1 min	1.2 sec
10,000 customers	6 min	5.4 sec
100,000 customers	40 min	< 1 min

Heuristic: Performance

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Heuristic Approximation Errors:

50 customers	1.9%
100 customers	0.85%
1,000 customers	0.08%

Application to the KAS

Personalized Estimates:

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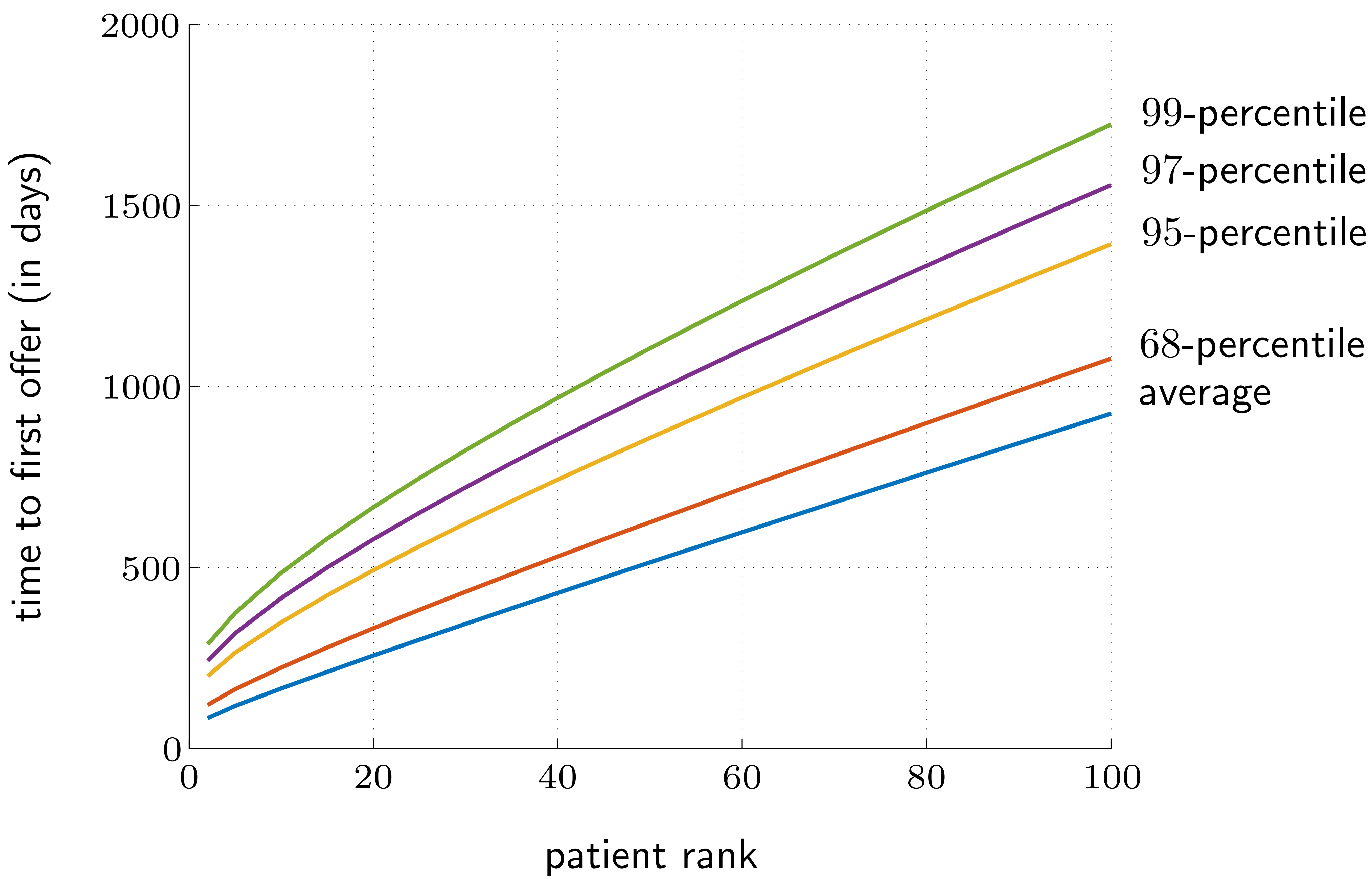
- PADV-OP1 Gift of Life Donor Program
- Threshold type decisions
- Model as HMCMS

Data & Approach

- Well accepted kidney quality metric: KDPI
- Historical kidney procurement rates (for each quality)
- Historical patient accept/decline decisions
- 2007-2010 training set
- 2010-2013 testing set

Out-of-Sample Performance



Out-of-Sample Performance

- Relative prediction errors
  - 14.96% for avg. and 11.73% for 68-percentile
- Delay history estimator:
  - Uses personalized info unavailable in practice
  - Cannot estimate wait times for high ranks
- Relative prediction errors of delay history estimator:
  - 16.76% for avg. and 14.65% for 68-percentile

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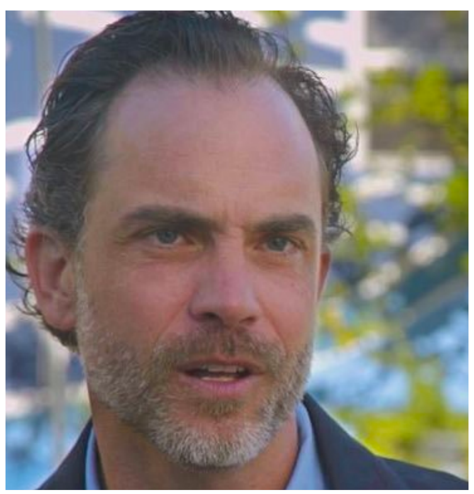
Partner

LAHSA
LOS ANGELES
HOMELESS
SERVICES
AUTHORITY

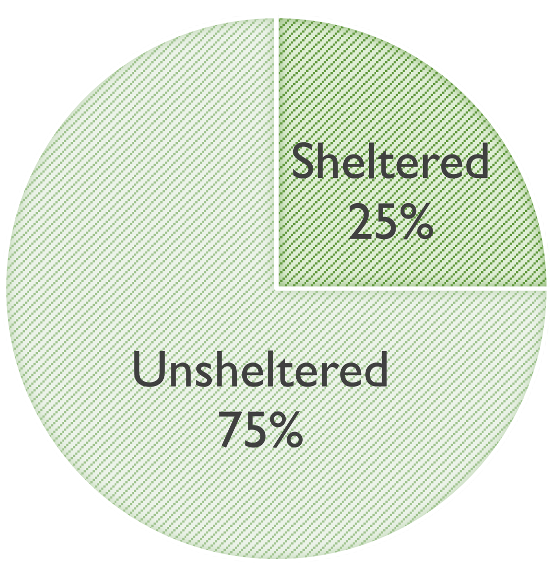


Eric Rice
CAIS Director
USC School of SW

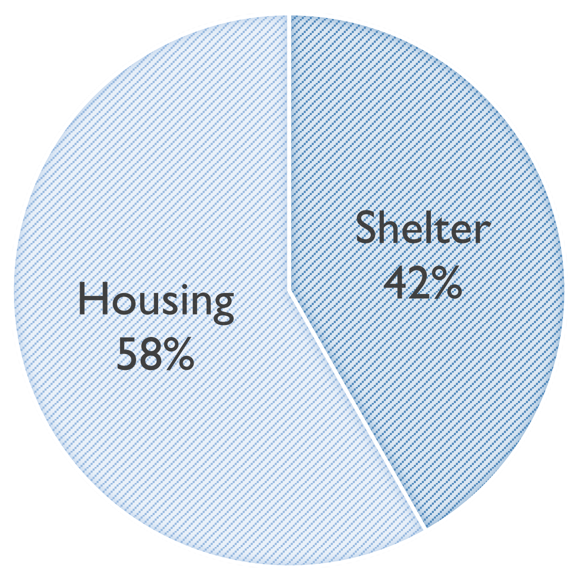
Homelessness Crisis

Homelessness Crisis

HOMELESS: TOTAL 50K



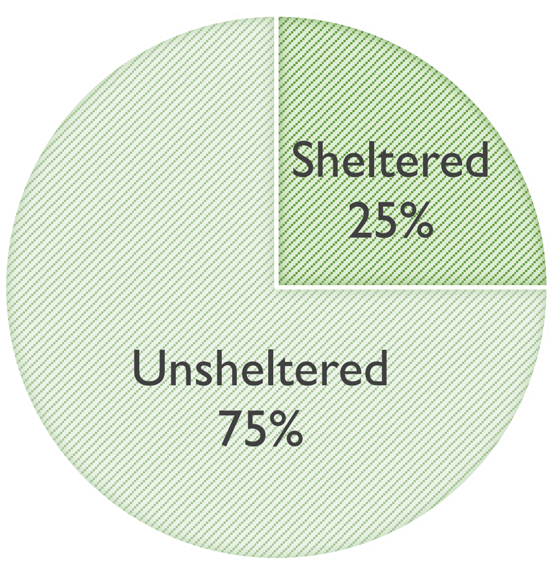
HOUSING: TOTAL 39K



LAHSA
LOS ANGELES
HOMELESS
SERVICES
AUTHORITY

Homelessness Crisis

HOMELESS: TOTAL 50K



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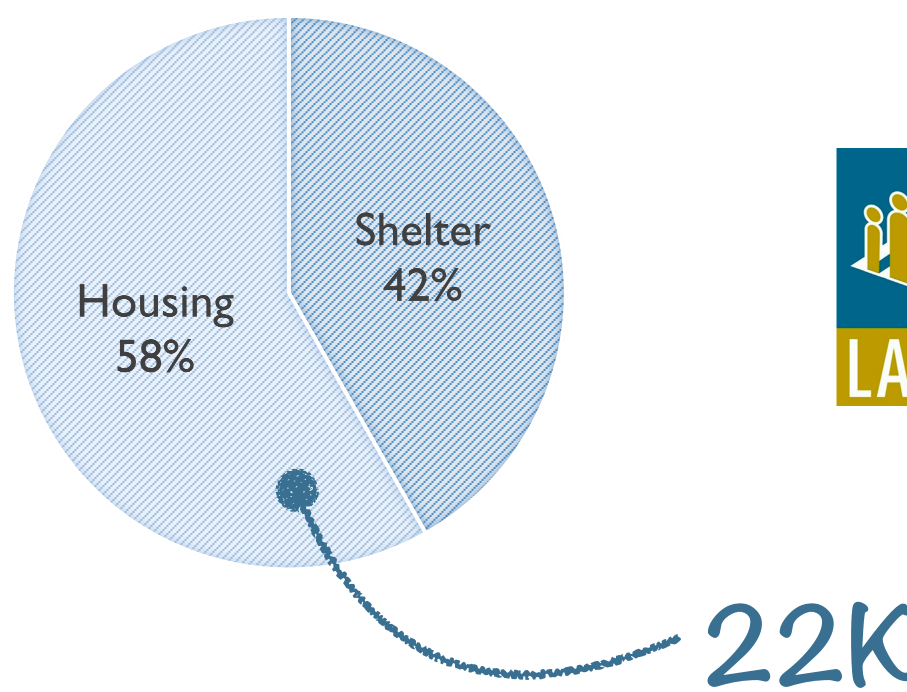
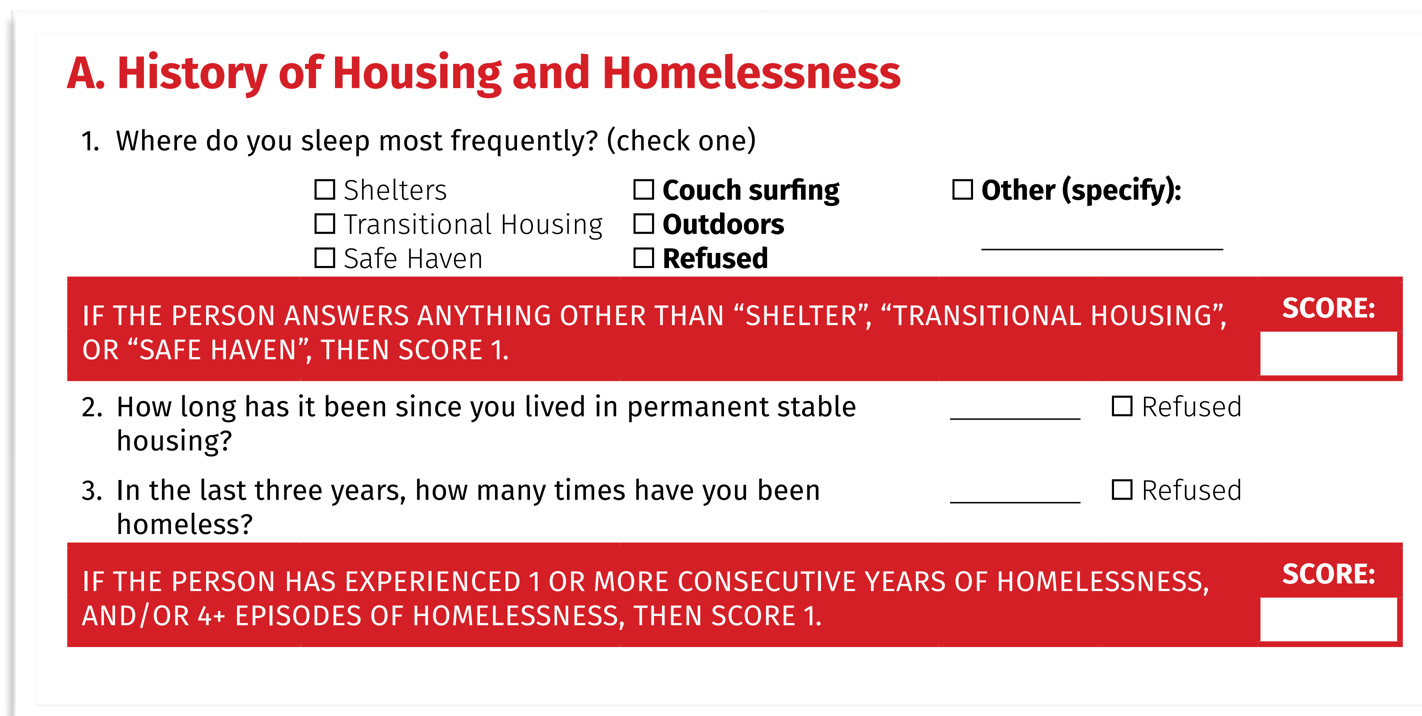


LOS ANGELES HOMELESS SERVICES AUTHORITY
LAHSA

Current Policy

Vulnerability Score



A. History of Housing and Homelessness

1. Where do you sleep most frequently? (check one)

☐ Shelters
☐ Transitional Housing
☐ Safe Haven
☐ **Couch surfing**
☐ **Outdoors**
☐ **Refused**
☐ **Other (specify):** ____________

IF THE PERSON ANSWERS ANYTHING OTHER THAN "SHELTER", "TRANSITIONAL HOUSING", OR "SAFE HAVEN", THEN SCORE 1. **SCORE:**

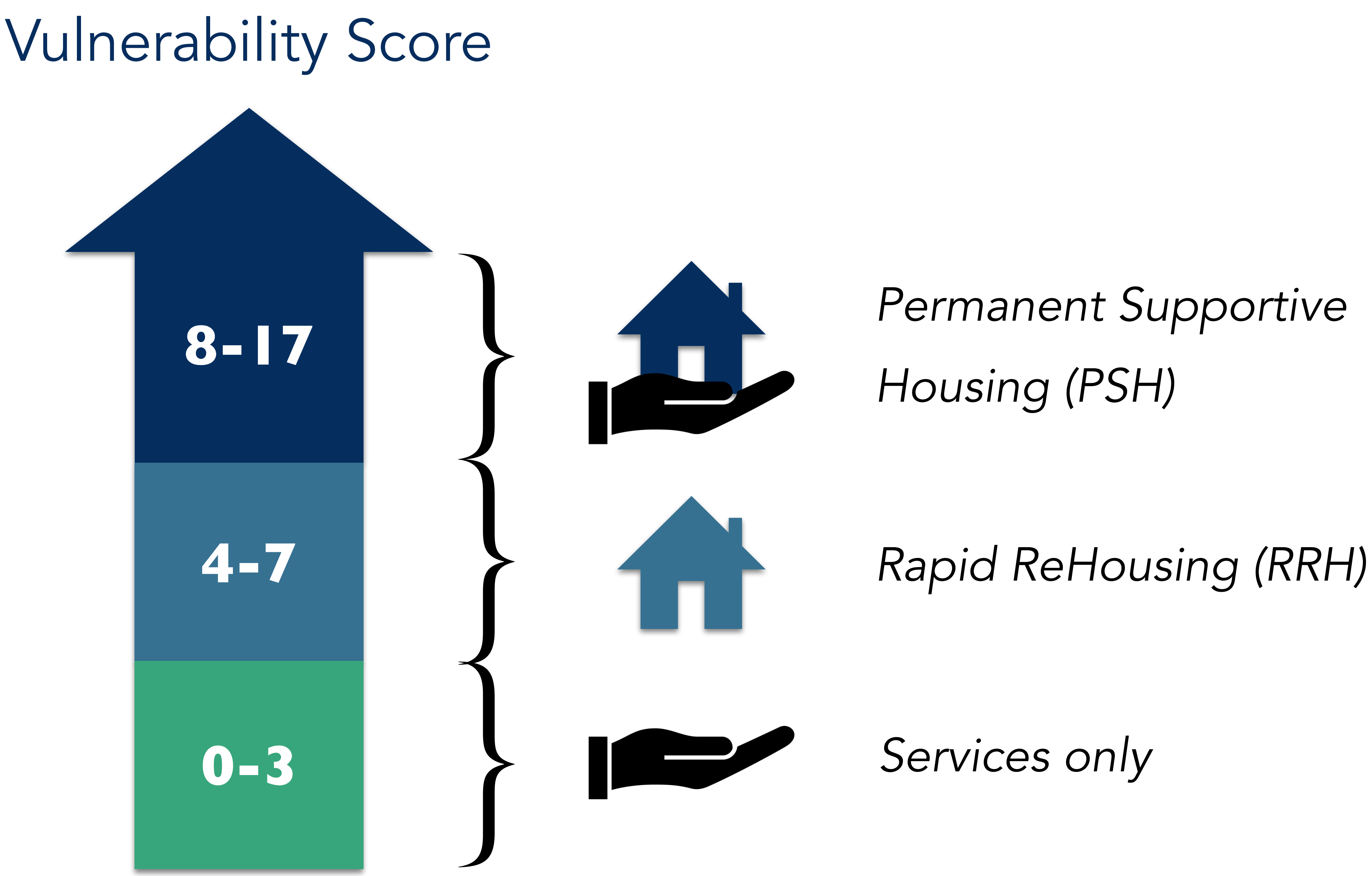
2. How long has it been since you lived in permanent stable housing? ________ ☐ Refused

3. In the last three years, how many times have you been homeless? ________ ☐ Refused

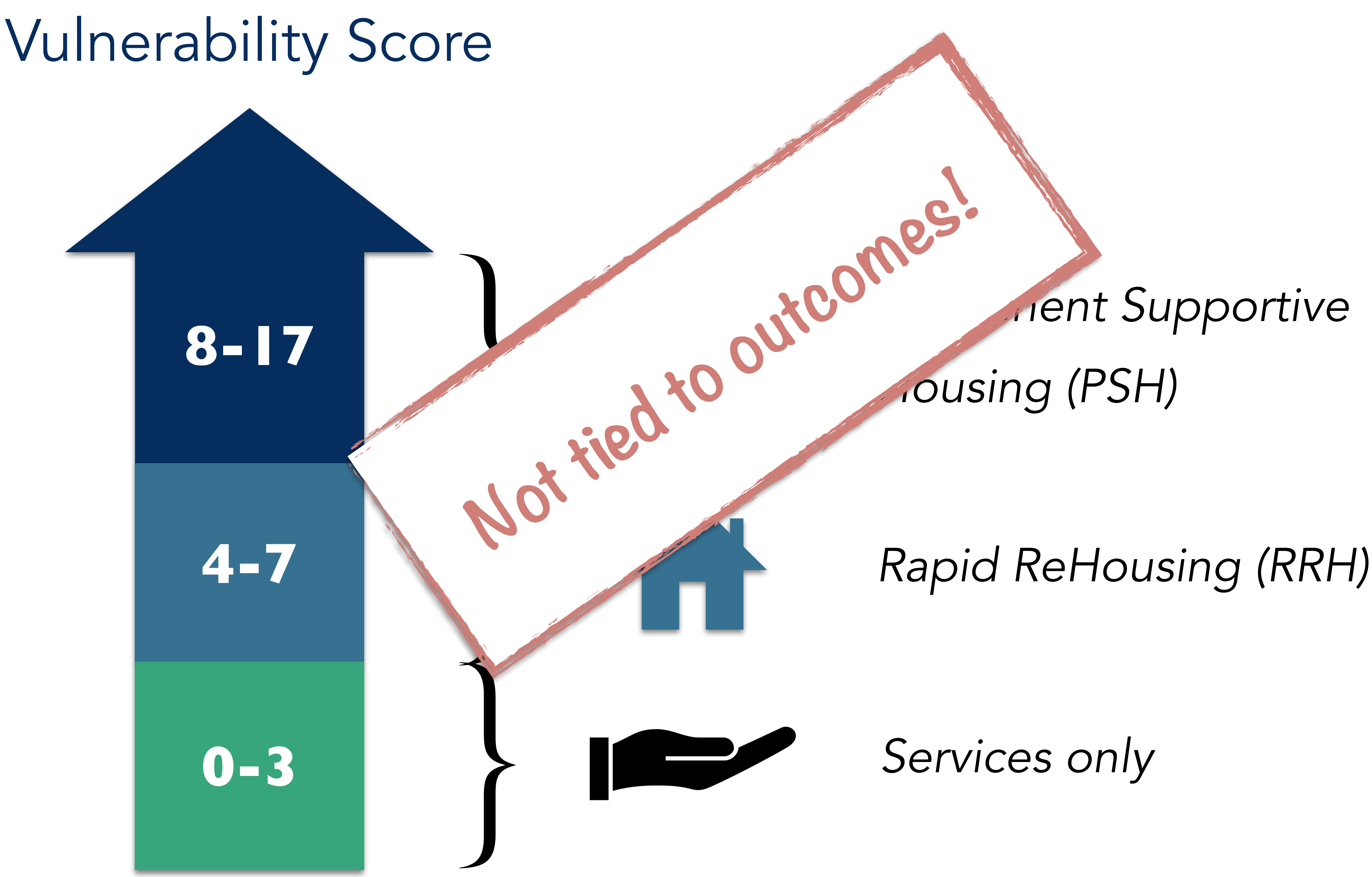
IF THE PERSON HAS EXPERIENCED 1 OR MORE CONSECUTIVE YEARS OF HOMELESSNESS, AND/OR 4+ EPISODES OF HOMELESSNESS, THEN SCORE 1. **SCORE:**

Vulnerability Calculated using Scoring Rule

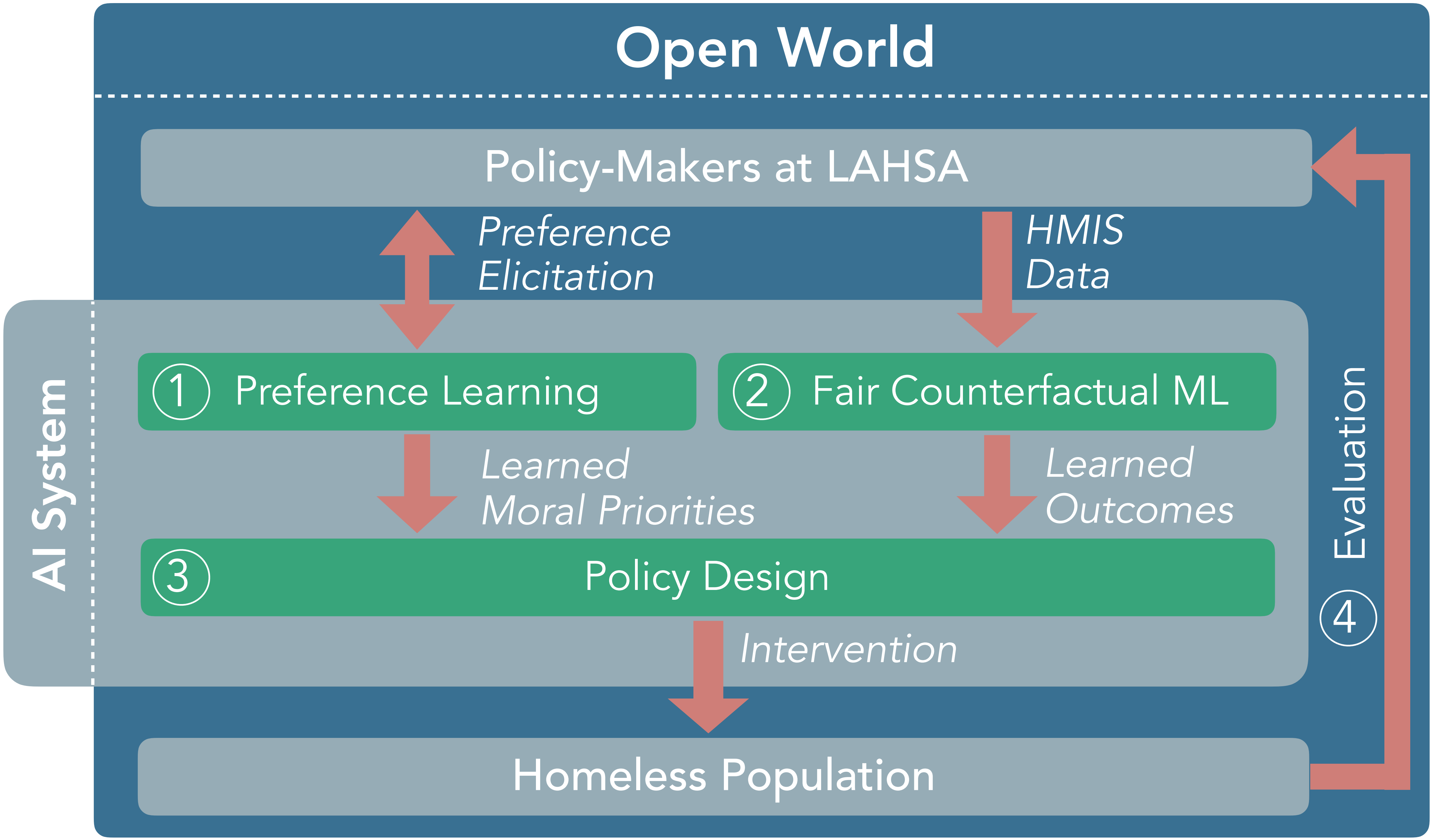
Current Policy



Current Policy



Proposed System



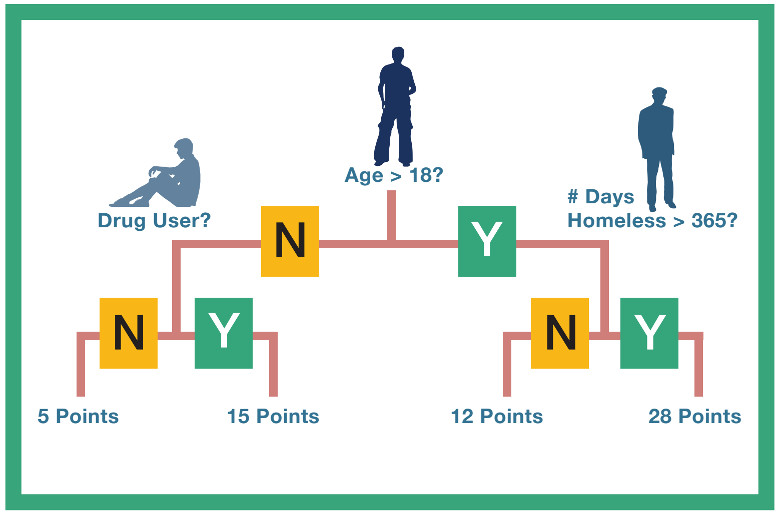
Outline

- ☑ Estimating Wait Times in Resource Allocation Systems
- ☐ Designing Policies for Allocating Scarce Resources
    - ☐ Preference Elicitation
    - ☐ Policy Optimization
- ☐ Optimizing "Gatekeeper Trainings" for Suicide Prevention

Policy Desiderata

Fairness

Efficiency

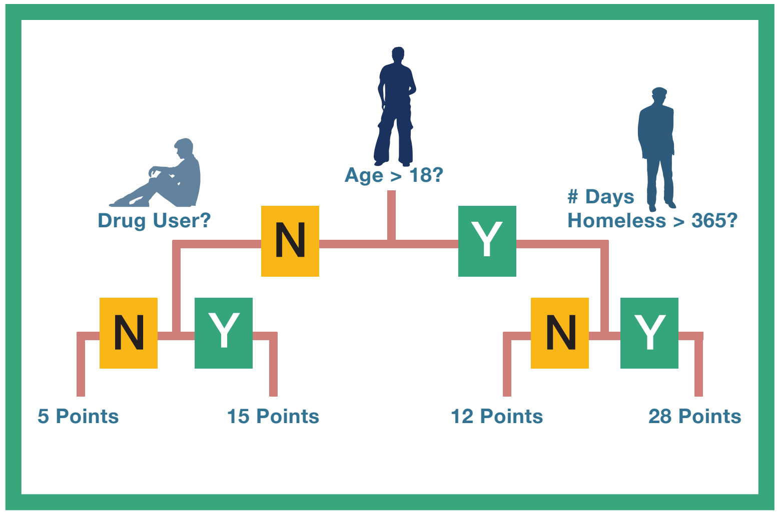


Interpretability

Policy Desiderata

Fairness

Efficiency

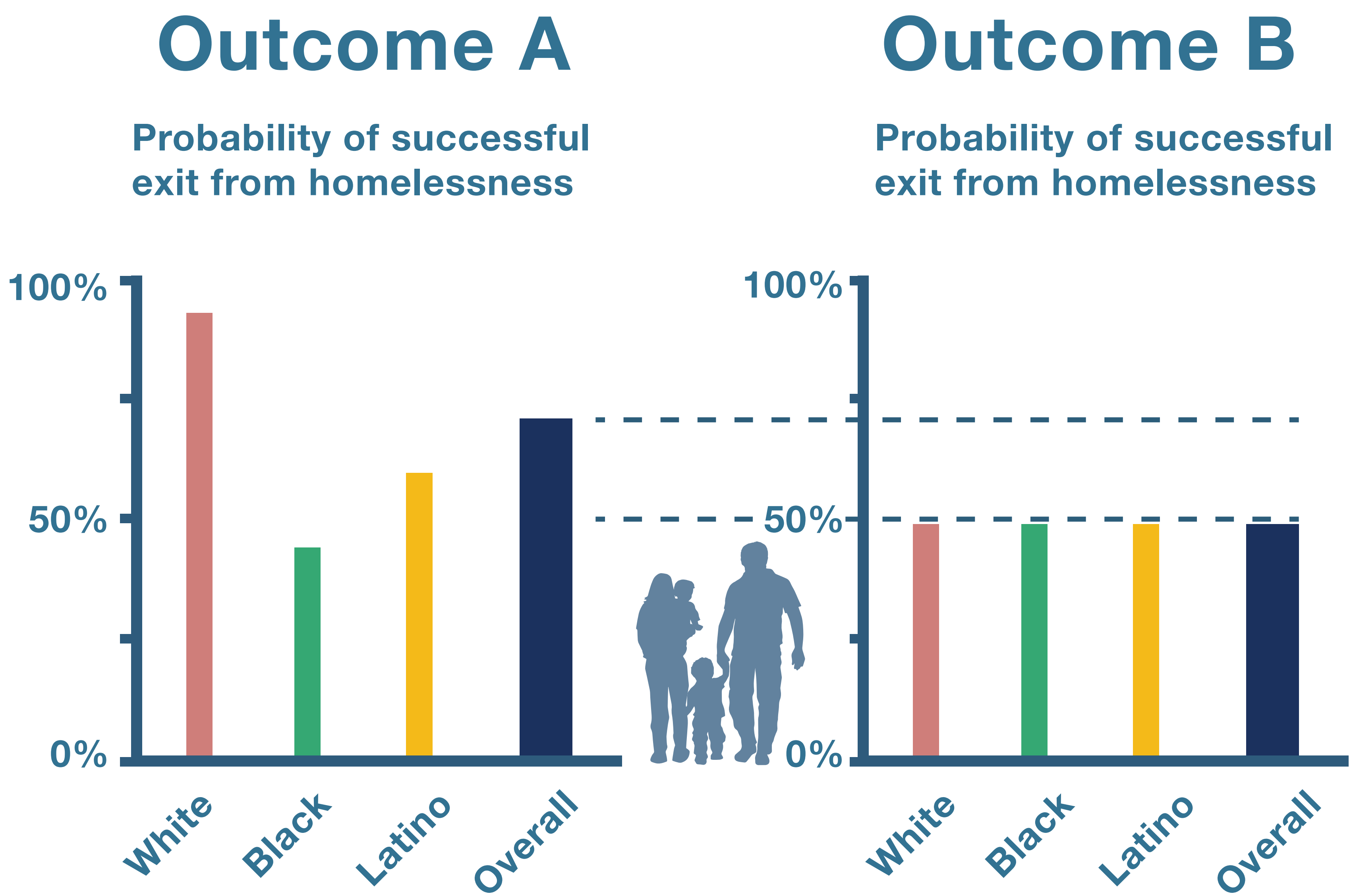


Interpretability

"This is the burning issue for us!"

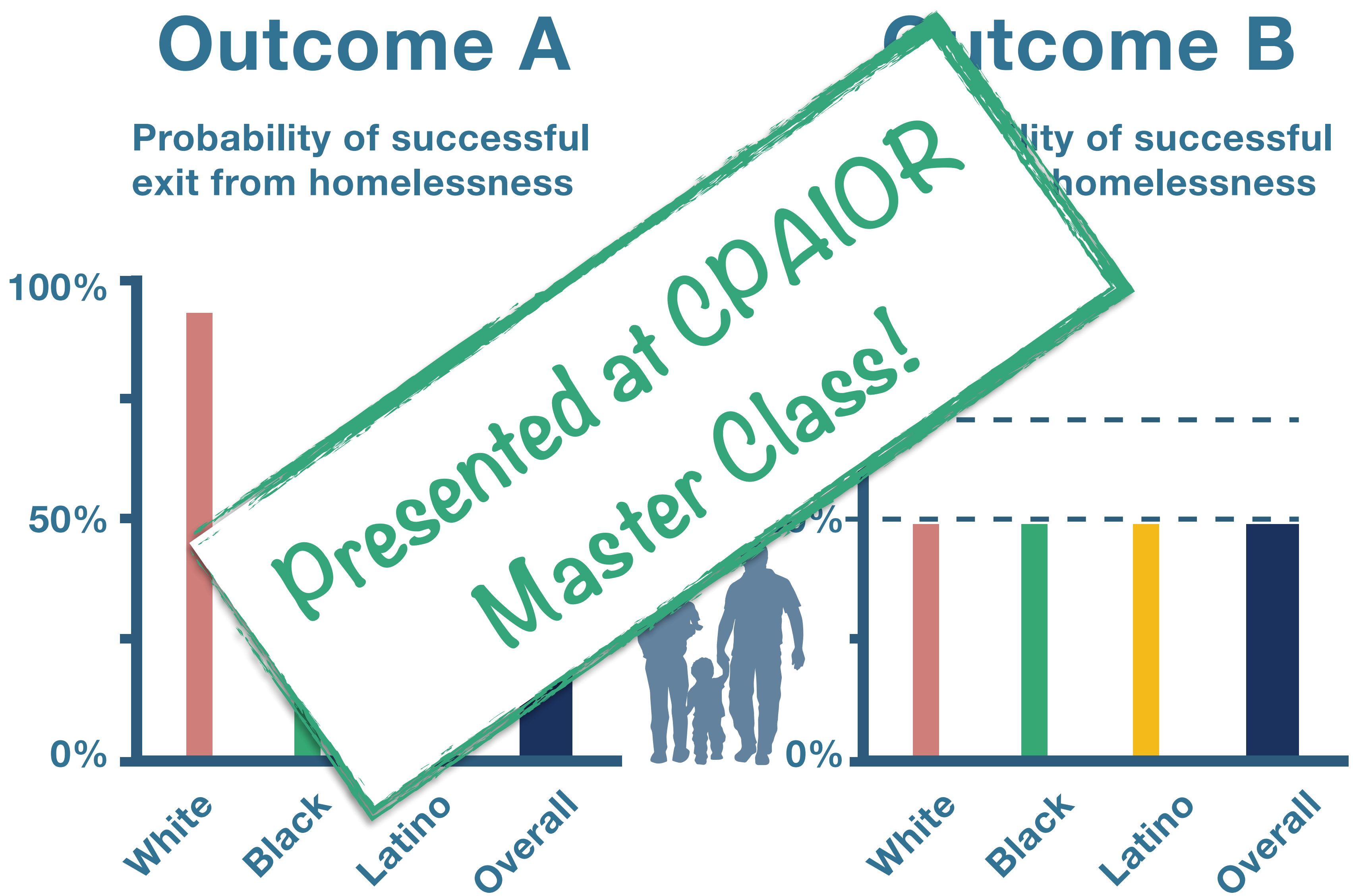
— Policy Supervisor at LAHSA —

Eliciting Moral Priorities



- Can ask pairwise comparisons:
  - "Do you prefer the policy A or policy B?"

Eliciting Moral Priorities



- Can ask pairwise comparisons:
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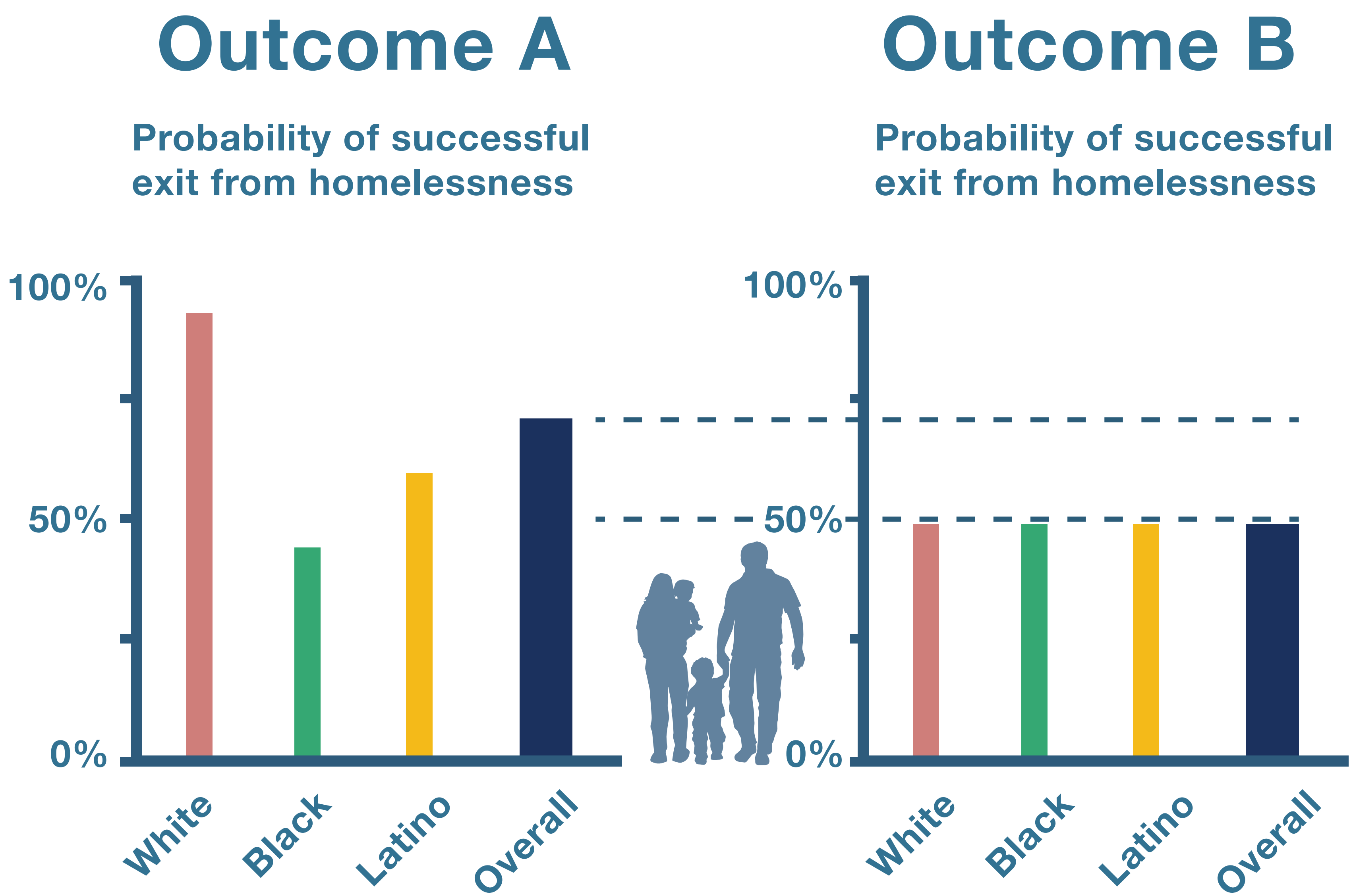
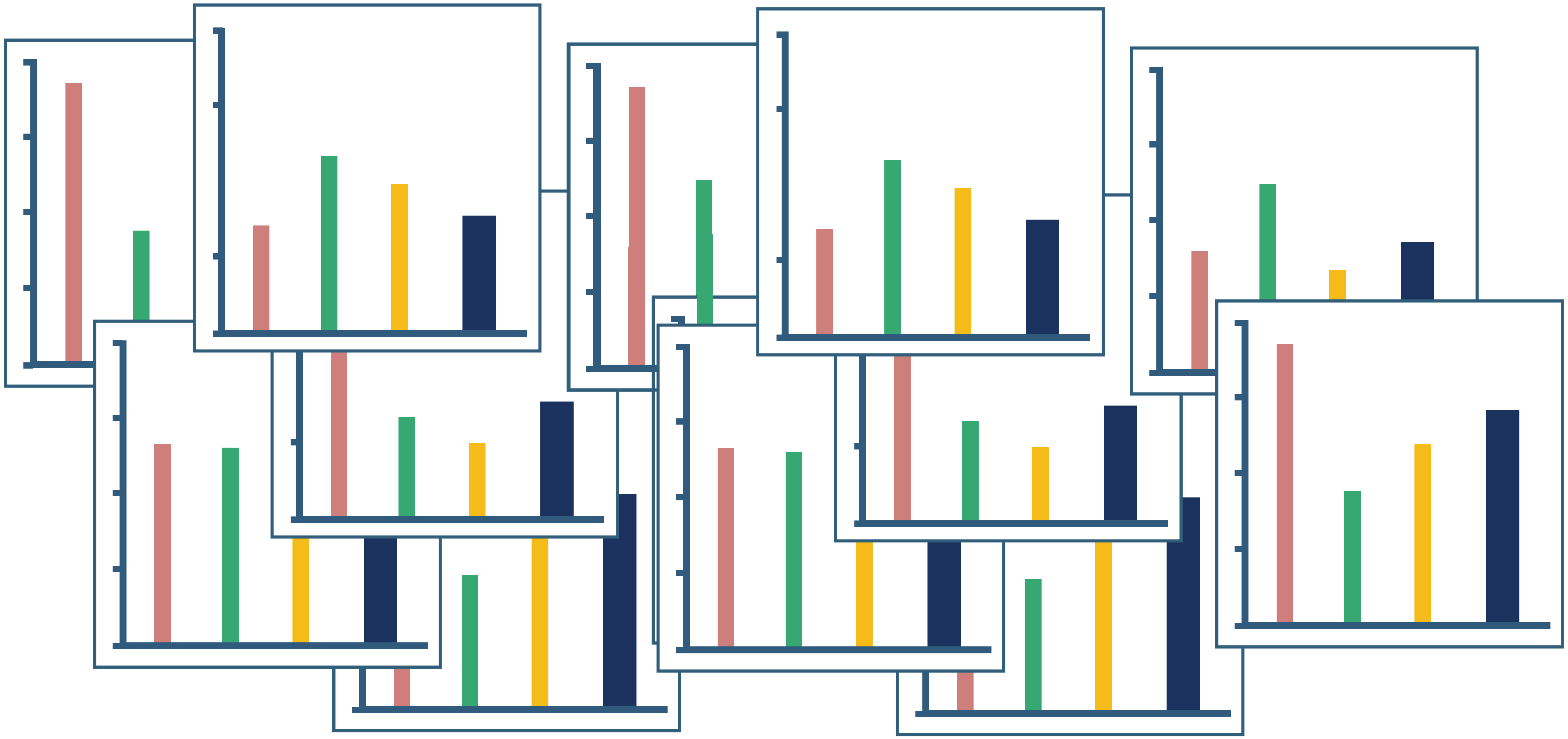
Eliciting Moral Priorities



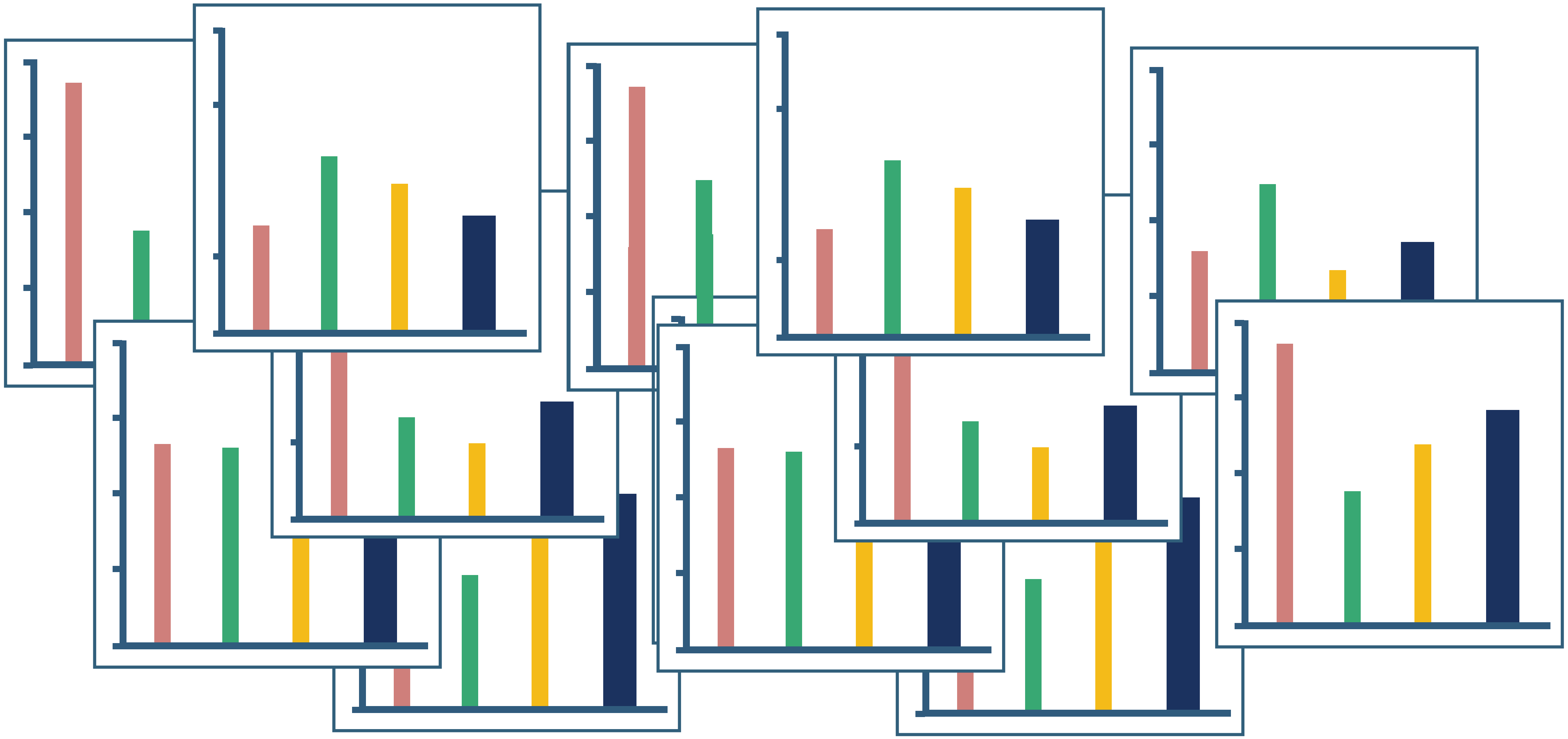
- Can ask how much they like a policy:

  - "How do you feel about policy A?"

Eliciting Moral Priorities

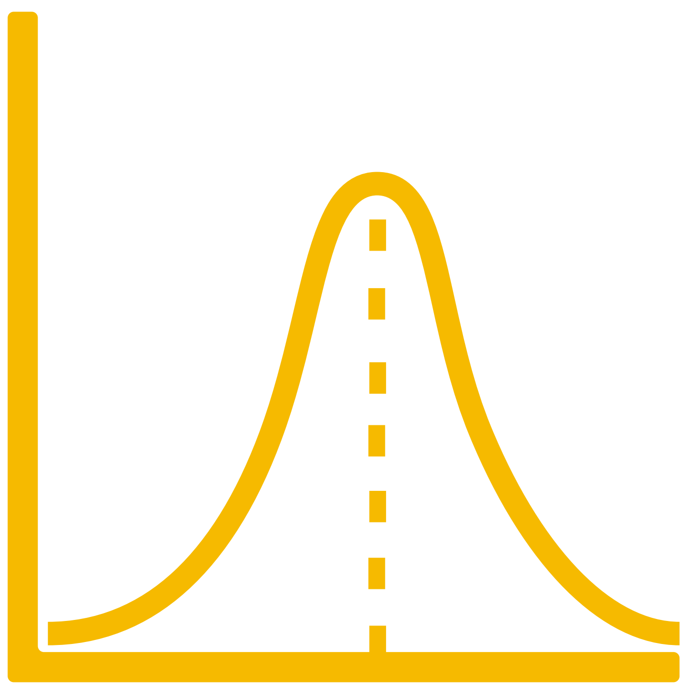


Eliciting Moral Priorities

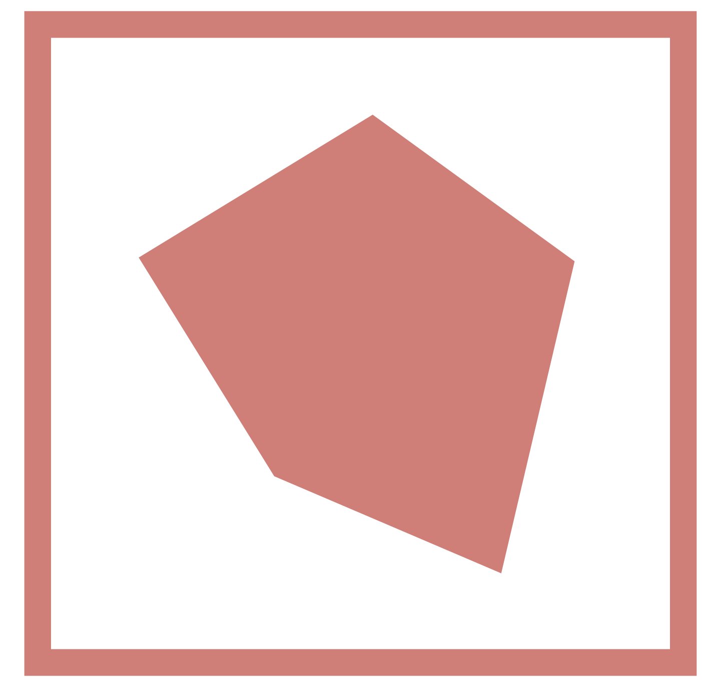


- Huge number of policies we can ask about
- Limited time (very under-resourced setting)
- Which questions to ask to gain the most useful information?

Robust Optimization

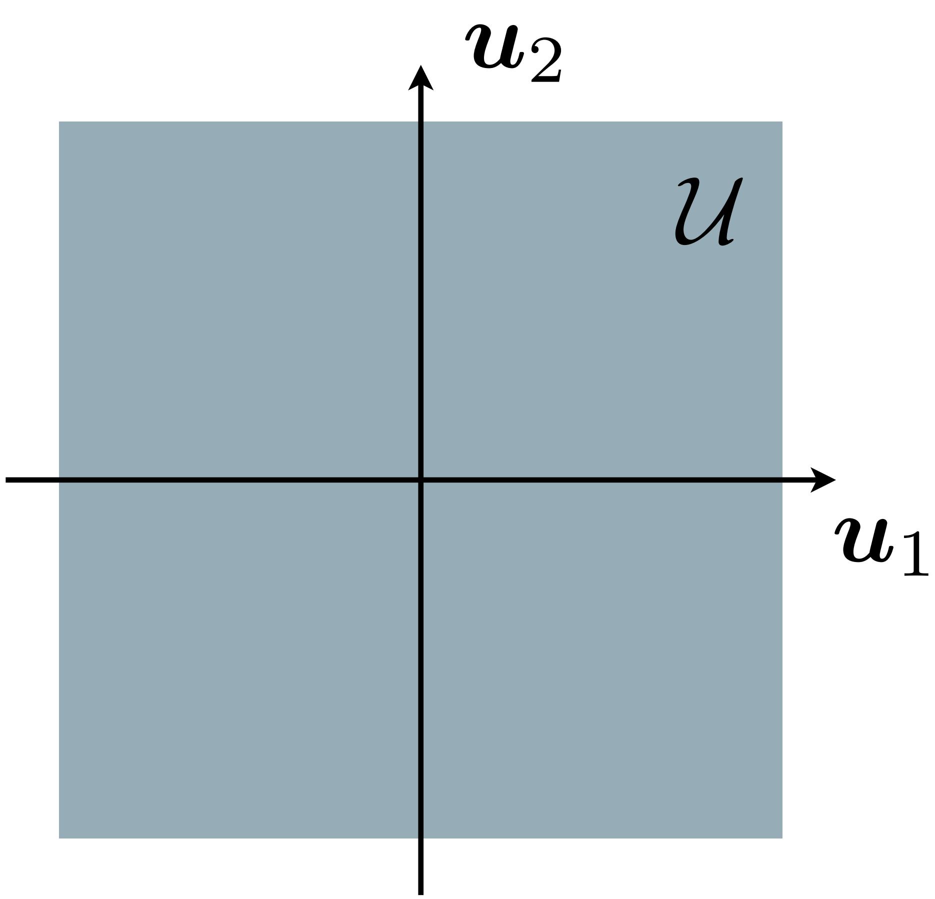


Distribution



Uncertainty Set

Eliciting Moral Priorities



Utility of policy with features $\boldsymbol{\phi}$

$$u(\boldsymbol{\phi}) = \boldsymbol{u}^\top \boldsymbol{\phi}$$

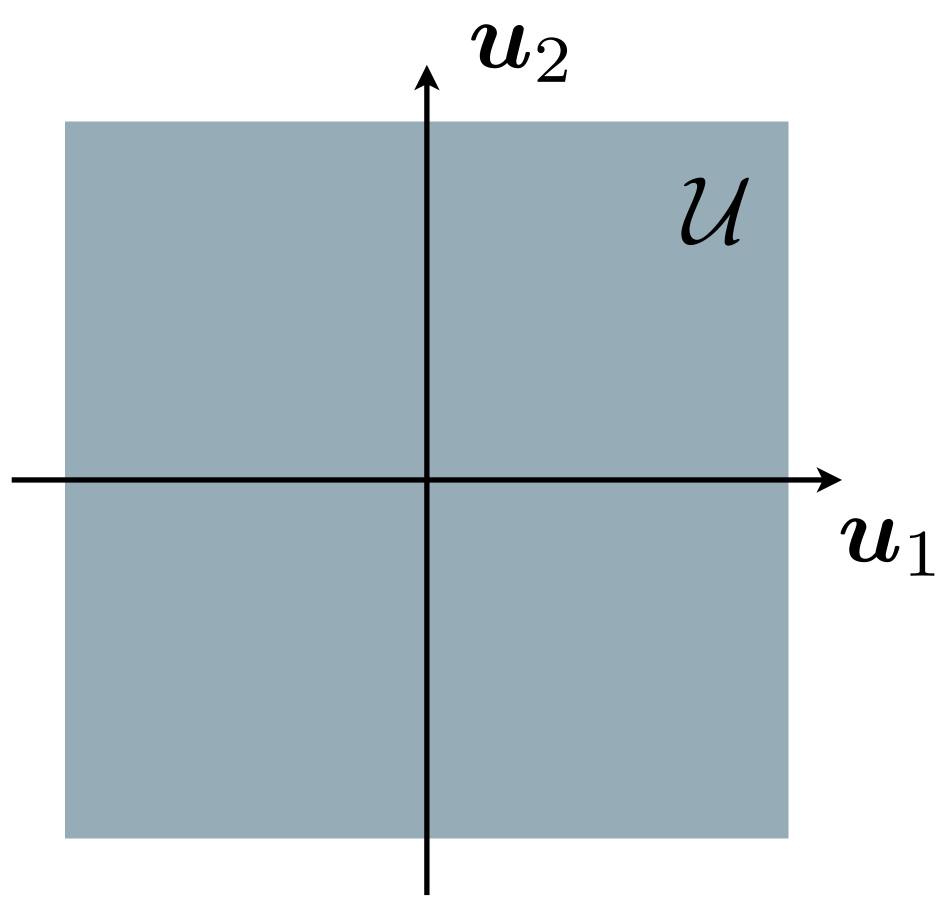
Features of a policy

Uncertain vector of utility function coefficients

Uncertainty Set

$$\Xi := \left\{ \boldsymbol{\xi} \in [0,1]^I \ : \ \exists \boldsymbol{u} \in [-1,1]^J \text{ such that } \boldsymbol{\xi}_i = \frac{\boldsymbol{u}^\top \boldsymbol{\phi}_i + \max_{j \in \mathcal{I}} \|\boldsymbol{\phi}_j\|_1}{2 \max_{j \in \mathcal{I}} \|\boldsymbol{\phi}_j\|_1} \ \forall i \in \mathcal{I} \right\}$$

Eliciting Moral Priorities



Utility of policy with features $\boldsymbol{\phi}$

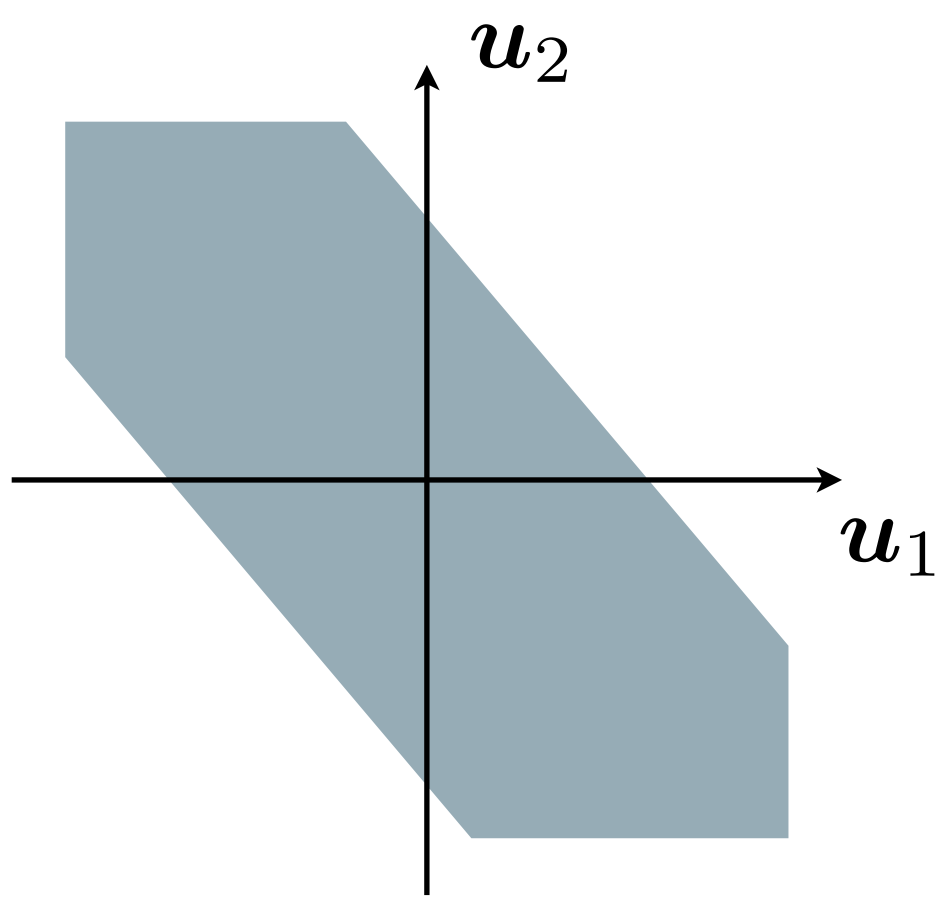
$$u(\boldsymbol{\phi}) = \boldsymbol{u}^\top \boldsymbol{\phi}$$

Features of a policy

Uncertain vector of utility function coefficients

- $\boldsymbol{u}$ is unknown and cannot be observed directly
- Answer $\boldsymbol{\xi}_i$ to question $i \in \mathcal{I}$ is unknown; only be revealed if we choose to spend some of our budget/time to ask that question

Eliciting Moral Priorities



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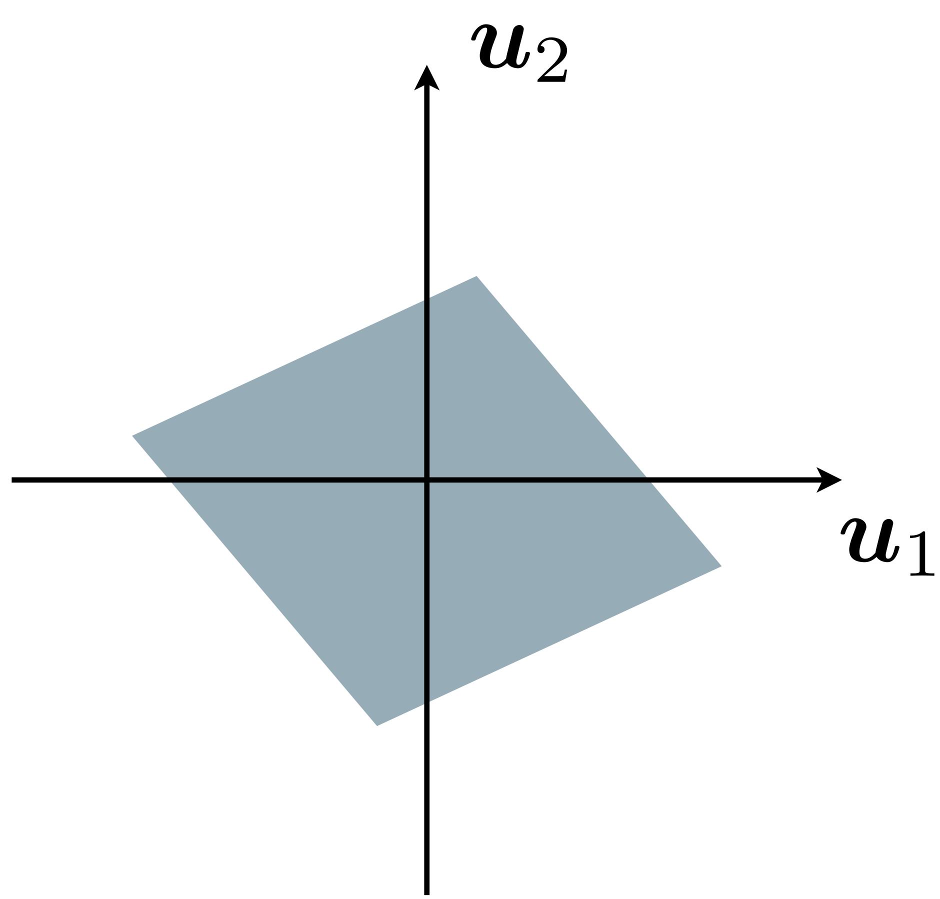

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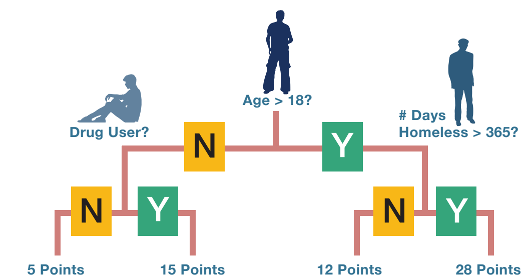
Static Elicitation



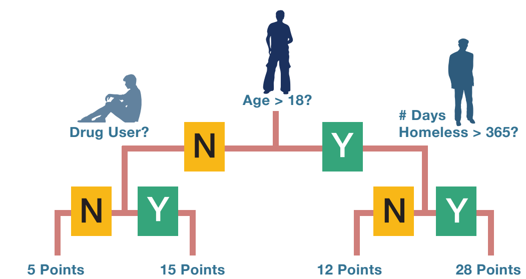
time

here-and-now:
decide which policies to ask about

observe:
answers to questions

wait-and-see:
offer a policy that meets the needs

Static Elicitation



time

here-and-now:

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observe:

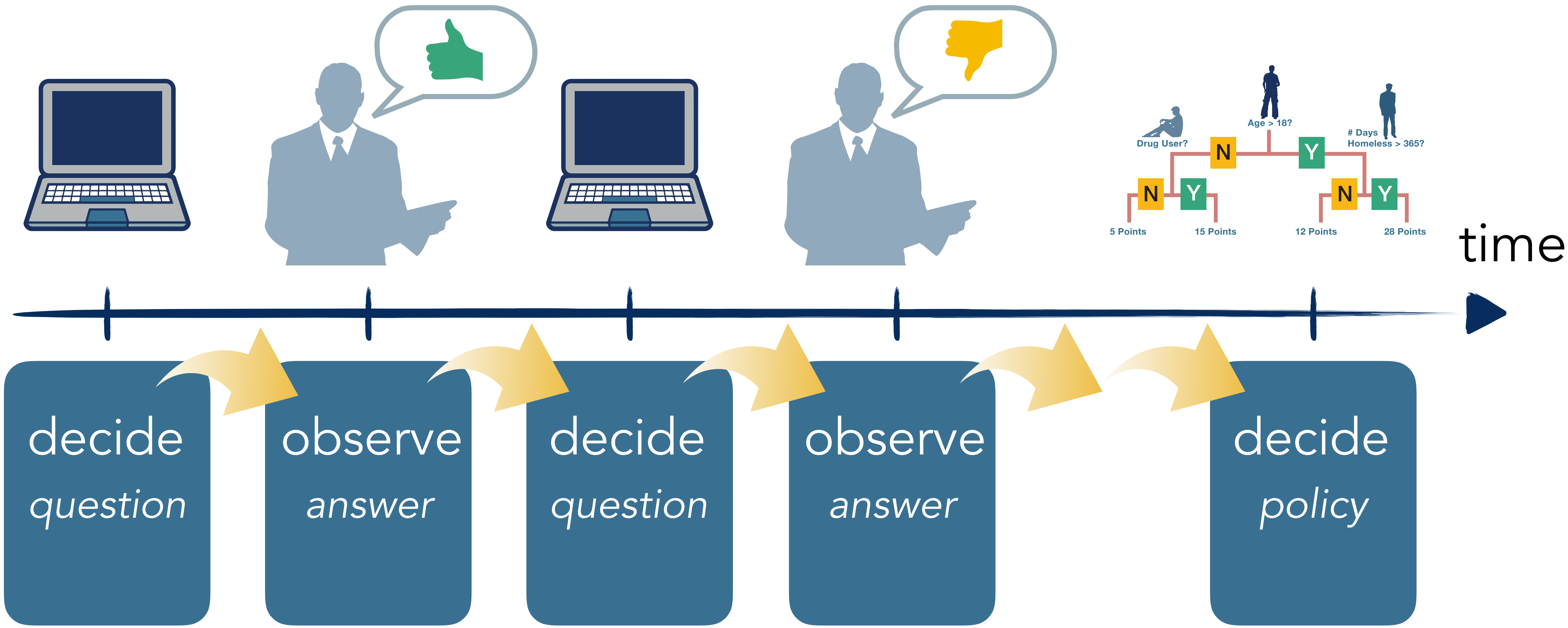
answers to questions

wait-and-see:

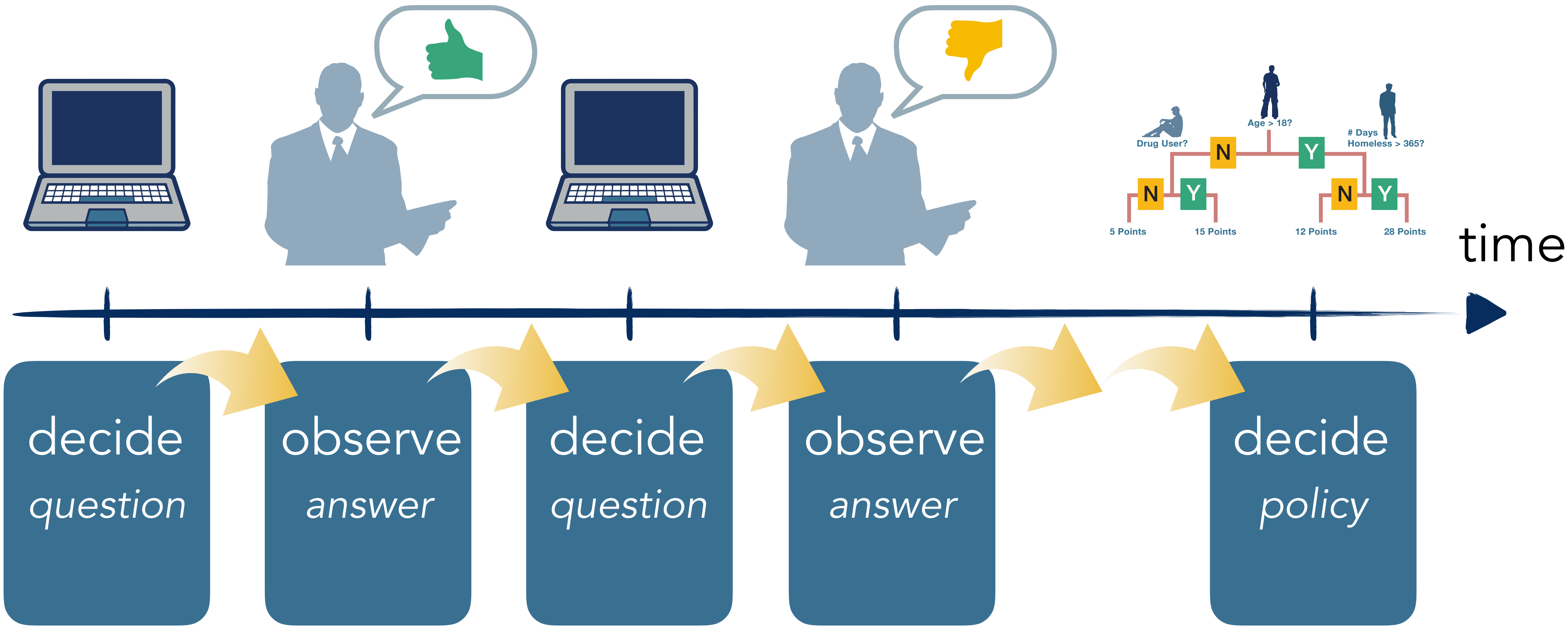
offer a policy that meets the needs

Two-Stage Robust Optimization with Decision-Dependent Information Discovery

Adaptive Elicitation



Adaptive Elicitation



Multi-Stage Robust Optimization with Decision-Dependent Information Discovery

Two-Stage RO with DDID

Sequence of Events:

here-and-now:

$\boldsymbol{x} \in \mathcal{X}$

$\boldsymbol{w} \in \mathcal{W} \subseteq \{0,1\}^{N_\xi}$

observe:

$\boldsymbol{\delta} = \boldsymbol{w} \circ \boldsymbol{\xi}$:

$\boldsymbol{\xi} \in \Xi$

wait-and-see:

$\boldsymbol{y} \in \mathcal{Y}$

Example:

Two-Stage RO with DDID

Sequence of Events:

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$\boldsymbol{x} \in \mathcal{X}$

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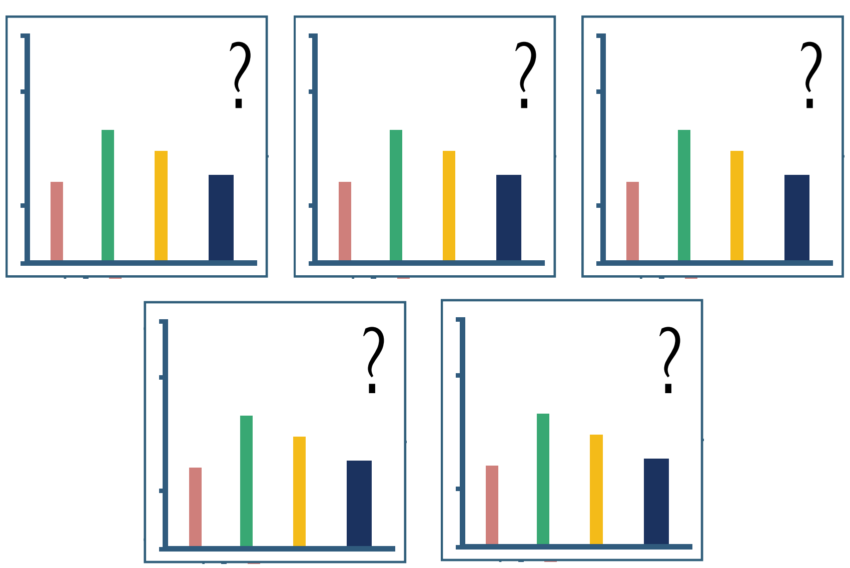

Two-Stage RO with DDID

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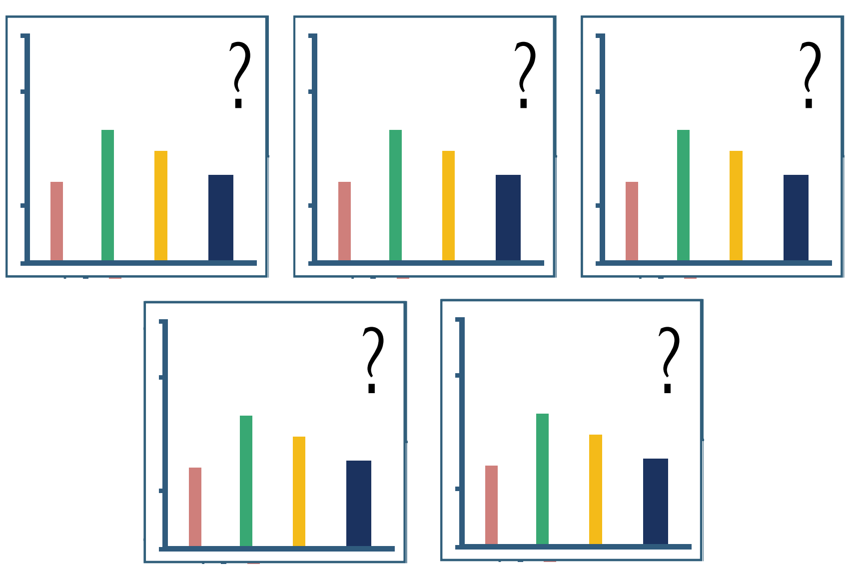
Example:



- $\boldsymbol{\xi}_i$: utility of policy i
- If no question asked:

$$\boldsymbol{\delta} = \boldsymbol{w} \circ \boldsymbol{\xi} = (0,0,0,0,0)$$

Two-Stage RO with DDID

Sequence of Events:

here-and-now:

$\boldsymbol{x} \in \mathcal{X}$

$\boldsymbol{w} \in \mathcal{W} \subseteq \{0,1\}^{N_\xi}$

observe:

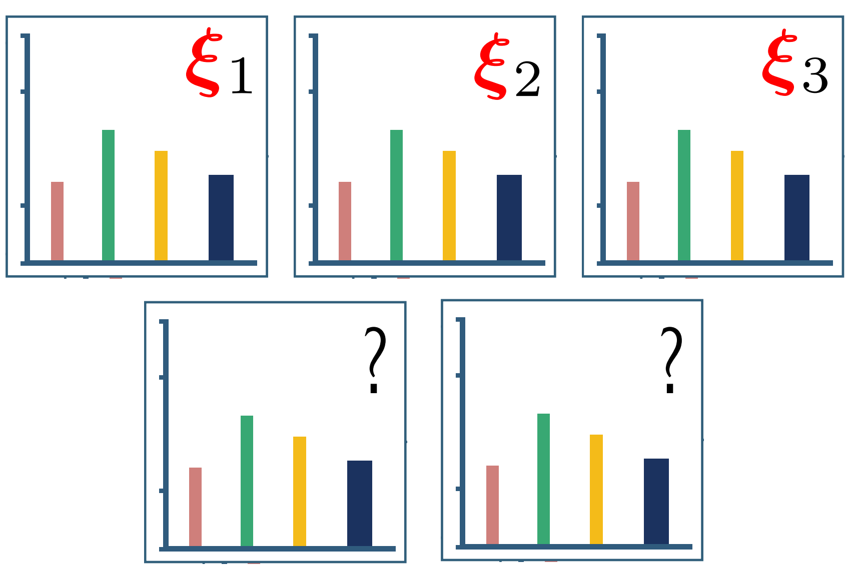
$\boldsymbol{\delta} = \boldsymbol{w} \circ \boldsymbol{\xi}$:

$\boldsymbol{\xi} \in \Xi$

wait-and-see:

$\boldsymbol{y} \in \mathcal{Y}$

Example:

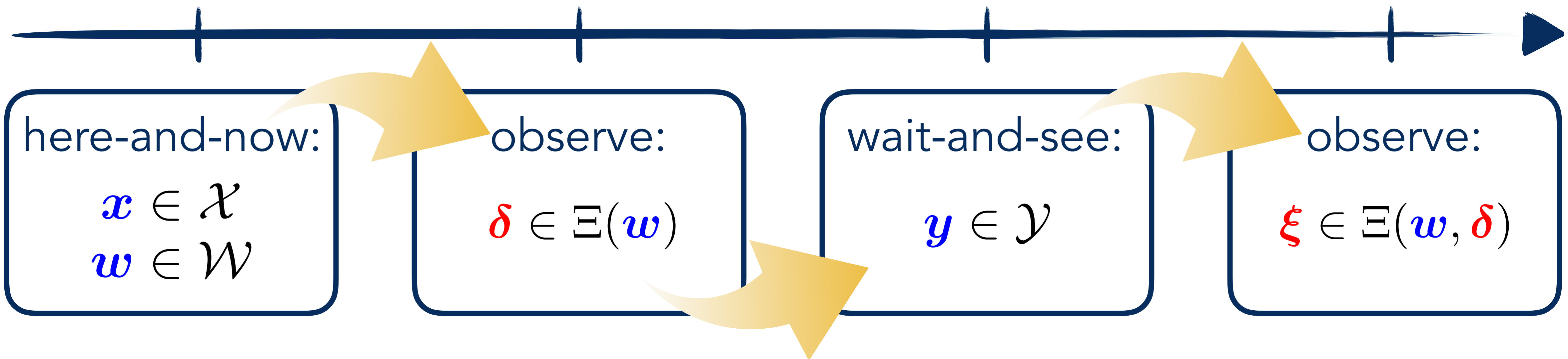


- $\boldsymbol{\xi}_i$: utility of policy i
- If ask utility of policies 1, 2, 3:

$$\boldsymbol{\delta} = \boldsymbol{w} \circ \boldsymbol{\xi} = (\boldsymbol{\xi}_1, \boldsymbol{\xi}_2, \boldsymbol{\xi}_3, 0, 0)$$

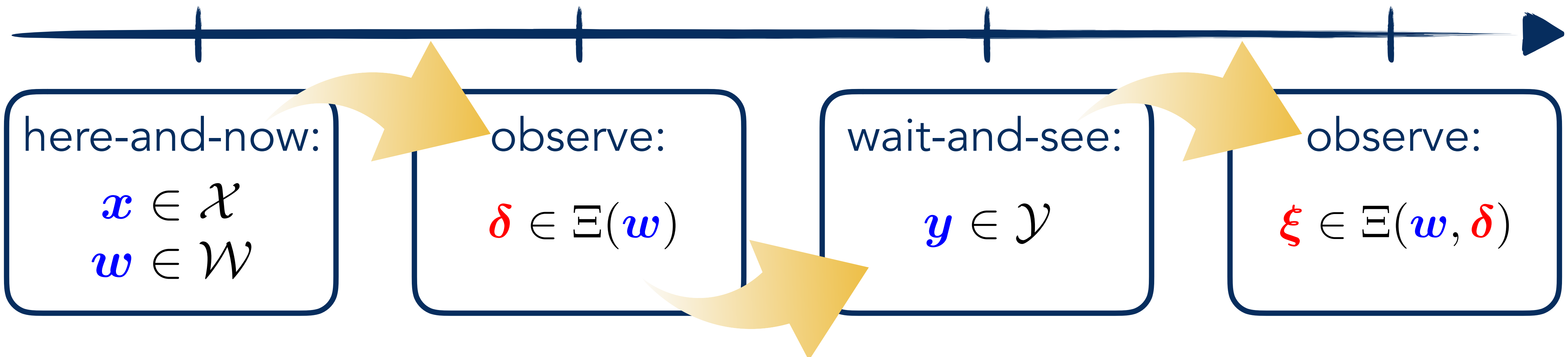
Two-Stage RO with DDID

Modeling with Dynamics:



Two-Stage RO with DDID

Modeling with Dynamics:

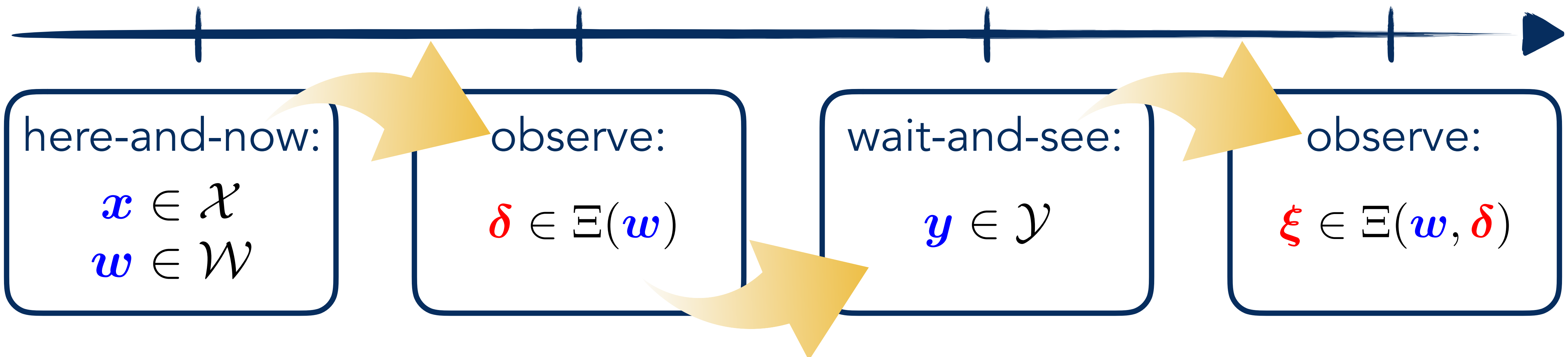


- Projection onto space of observed uncertainties:

$$\Xi(\boldsymbol{w}) = \{\boldsymbol{\delta} \in \mathbb{R}^{N_\xi} \ : \ \exists \boldsymbol{\xi} \in \Xi \text{ with } \boldsymbol{\delta} = \boldsymbol{w} \circ \boldsymbol{\xi}\}$$

Two-Stage RO with DDID

Modeling with Dynamics:



- Projection onto space of observed uncertainties:

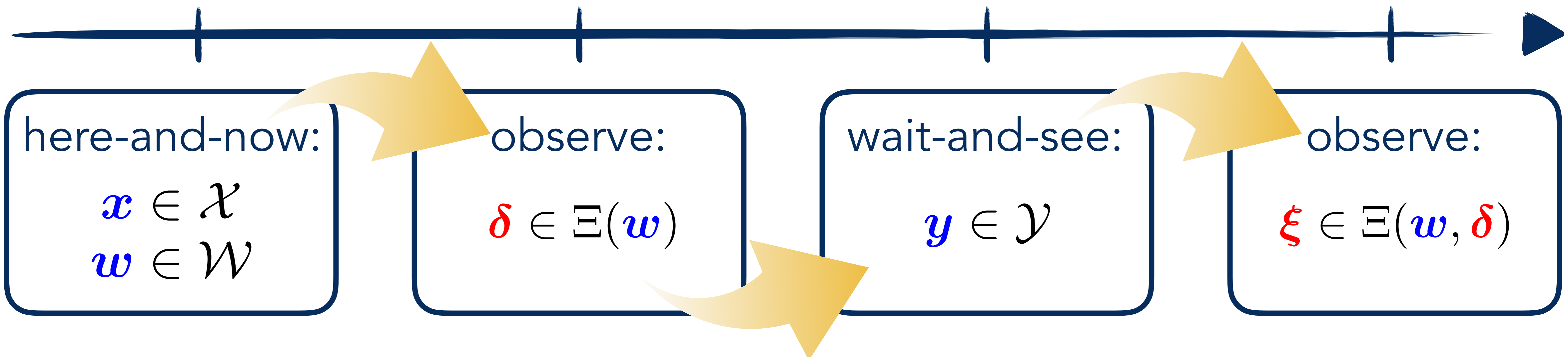
$$\Xi(\boldsymbol{w}) = \{\boldsymbol{\delta} \in \mathbb{R}^{N_\xi} \ : \ \exists \boldsymbol{\xi} \in \Xi \text{ with } \boldsymbol{\delta} = \boldsymbol{w} \circ \boldsymbol{\xi}\}$$

- Subset compatible with observed uncertainties:

$$\Xi(\boldsymbol{w}, \boldsymbol{\delta}) = \{\boldsymbol{\xi} \in \Xi \ : \ \boldsymbol{w} \circ \boldsymbol{\xi} = \boldsymbol{w} \circ \boldsymbol{\delta}\}$$

Two-Stage RO with DDID

Modeling with Dynamics:

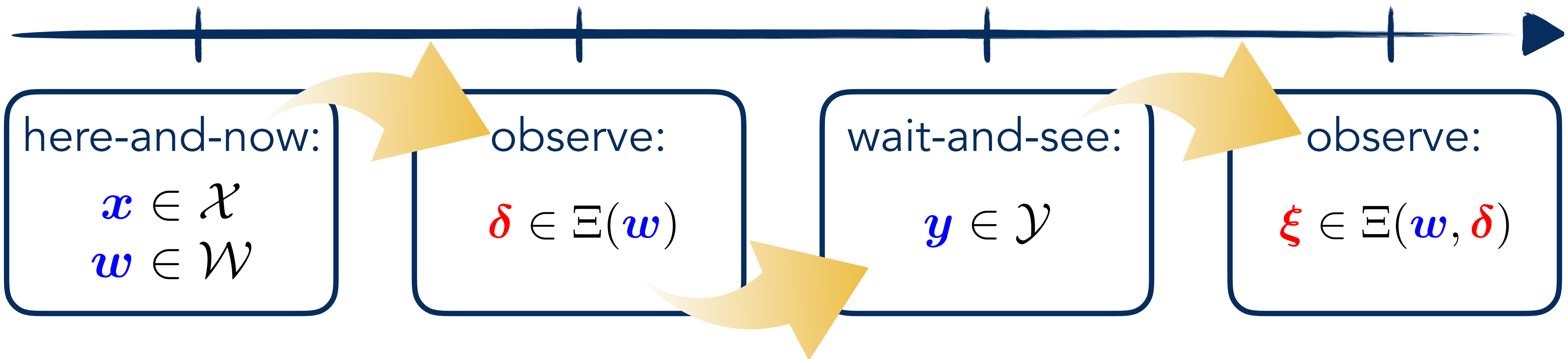


Problem Formulation

$$
\begin{array}{llll}
\underset{\boldsymbol{x}\in\mathcal{X},\, \boldsymbol{w}\in\mathcal{W}}{\text{minimize}} & \underset{\boldsymbol{\delta}\in\Xi(\boldsymbol{w})}{\max} & \underset{\boldsymbol{y}\in\mathcal{Y}}{\min} & \underset{\boldsymbol{\xi}\in\Xi(\boldsymbol{w},\boldsymbol{\delta})}{\max} \quad \boldsymbol{\xi}^\top \boldsymbol{C}\boldsymbol{x} + \boldsymbol{\xi}^\top \boldsymbol{D}\boldsymbol{w} + \boldsymbol{\xi}^\top \boldsymbol{Q}\boldsymbol{y} \\
 & & \text{s.t.} & \boldsymbol{T}\boldsymbol{x} + \boldsymbol{V}\boldsymbol{w} + \boldsymbol{W}\boldsymbol{y} + \leq \boldsymbol{h}(\boldsymbol{\xi}) \quad \forall \boldsymbol{\xi} \in \Xi(\boldsymbol{w}, \boldsymbol{\delta})
\end{array}
$$

Two-Stage RO with DDID

Modeling with Dynamics:



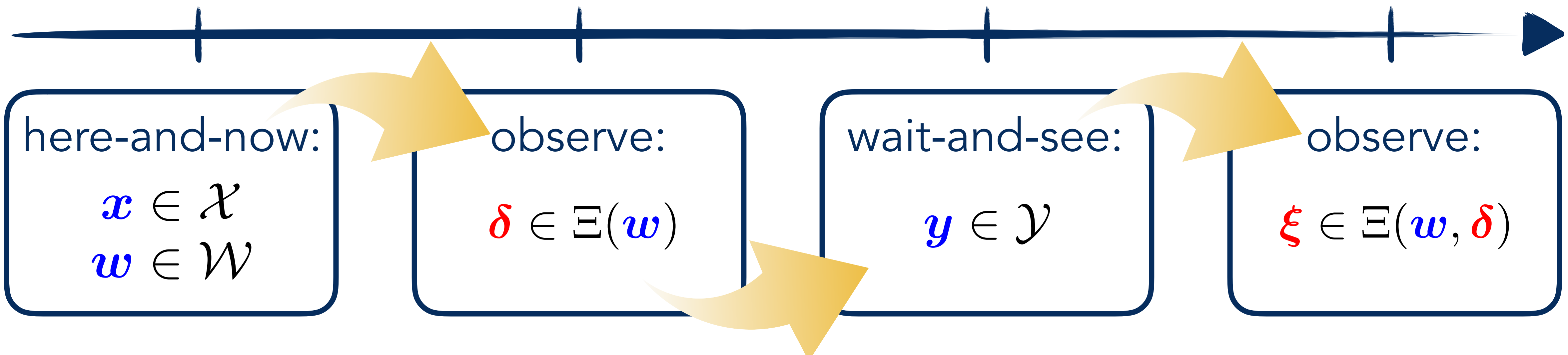
NP-Hard!

Problem Formulation

$$\begin{array}{llll} \underset{\boldsymbol{x}\in\mathcal{X},\ \boldsymbol{w}\in\mathcal{W}}{\text{minimize}} & \underset{\boldsymbol{\delta}\in\Xi(\boldsymbol{w})}{\max} & \underset{\boldsymbol{y}\in\mathcal{Y}}{\min} & \underset{\boldsymbol{\xi}\in\Xi(\boldsymbol{w},\boldsymbol{\delta})}{\max} \quad \boldsymbol{\xi}^\top \boldsymbol{C}\boldsymbol{x} + \boldsymbol{\xi}^\top \boldsymbol{D}\boldsymbol{w} + \boldsymbol{\xi}^\top \boldsymbol{Q}\boldsymbol{y} \\ & & \text{s.t.} & \boldsymbol{T}\boldsymbol{x} + \boldsymbol{V}\boldsymbol{w} + \boldsymbol{W}\boldsymbol{y} + \leq \boldsymbol{h}(\boldsymbol{\xi}) \quad \forall \boldsymbol{\xi} \in \Xi(\boldsymbol{w}, \boldsymbol{\delta}) \end{array}$$

Two-Stage RO with DDID

Modeling with Dynamics:

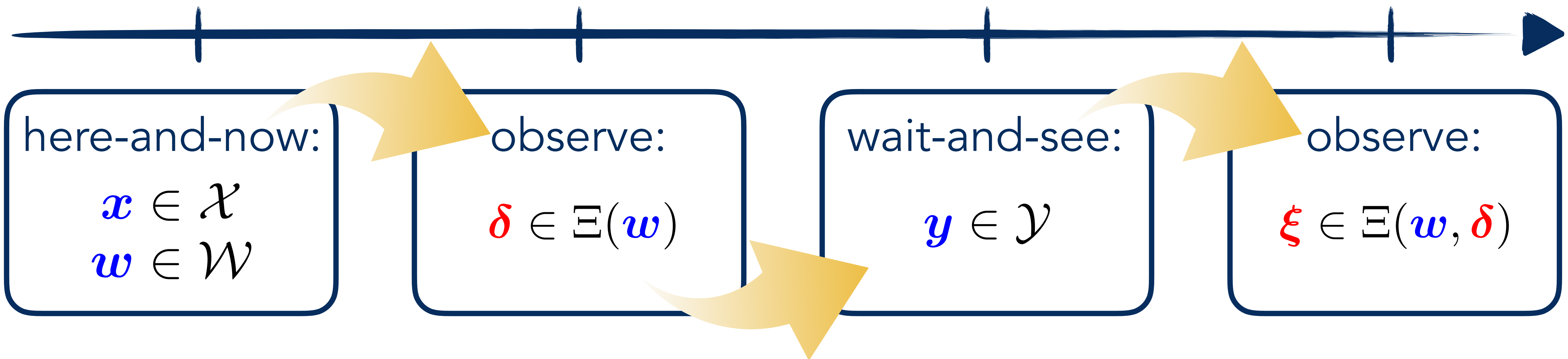


Correct!

Problem Formulation

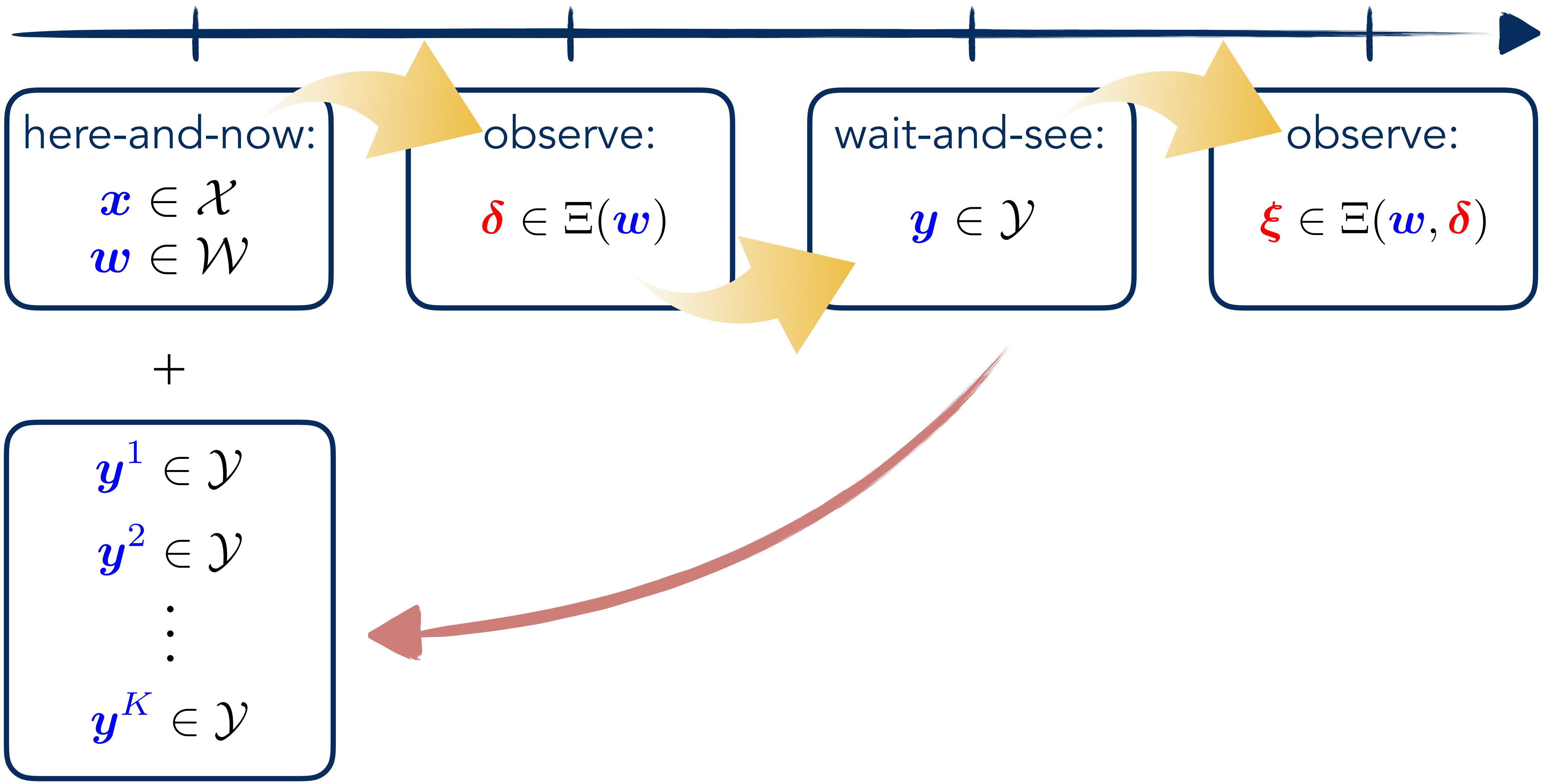
$$
\begin{array}{llll}
\underset{\boldsymbol{x}\in\mathcal{X},\, \boldsymbol{w}\in\mathcal{W}}{\text{minimize}} & \underset{\boldsymbol{\delta}\in\Xi(\boldsymbol{w})}{\max} & \underset{\boldsymbol{y}\in\mathcal{Y}}{\min} & \underset{\boldsymbol{\xi}\in\Xi(\boldsymbol{w},\boldsymbol{\delta})}{\max} \quad \boldsymbol{\xi}^\top \boldsymbol{C}\boldsymbol{x} + \boldsymbol{\xi}^\top \boldsymbol{D}\boldsymbol{w} + \boldsymbol{\xi}^\top \boldsymbol{Q}\boldsymbol{y} \\
 & & \text{s.t.} & \boldsymbol{T}\boldsymbol{x} + \boldsymbol{V}\boldsymbol{w} + \boldsymbol{W}\boldsymbol{y} + \leq \boldsymbol{h}(\boldsymbol{\xi}) \quad \forall \boldsymbol{\xi} \in \Xi(\boldsymbol{w}, \boldsymbol{\delta})
\end{array}
$$

RO with DDID: K-Adaptability



[1] Exogenous uncertainty: Hanasusanto et. al (2015), Caramanis and Bertsimas (2010)

RO with DDID: K-Adaptability



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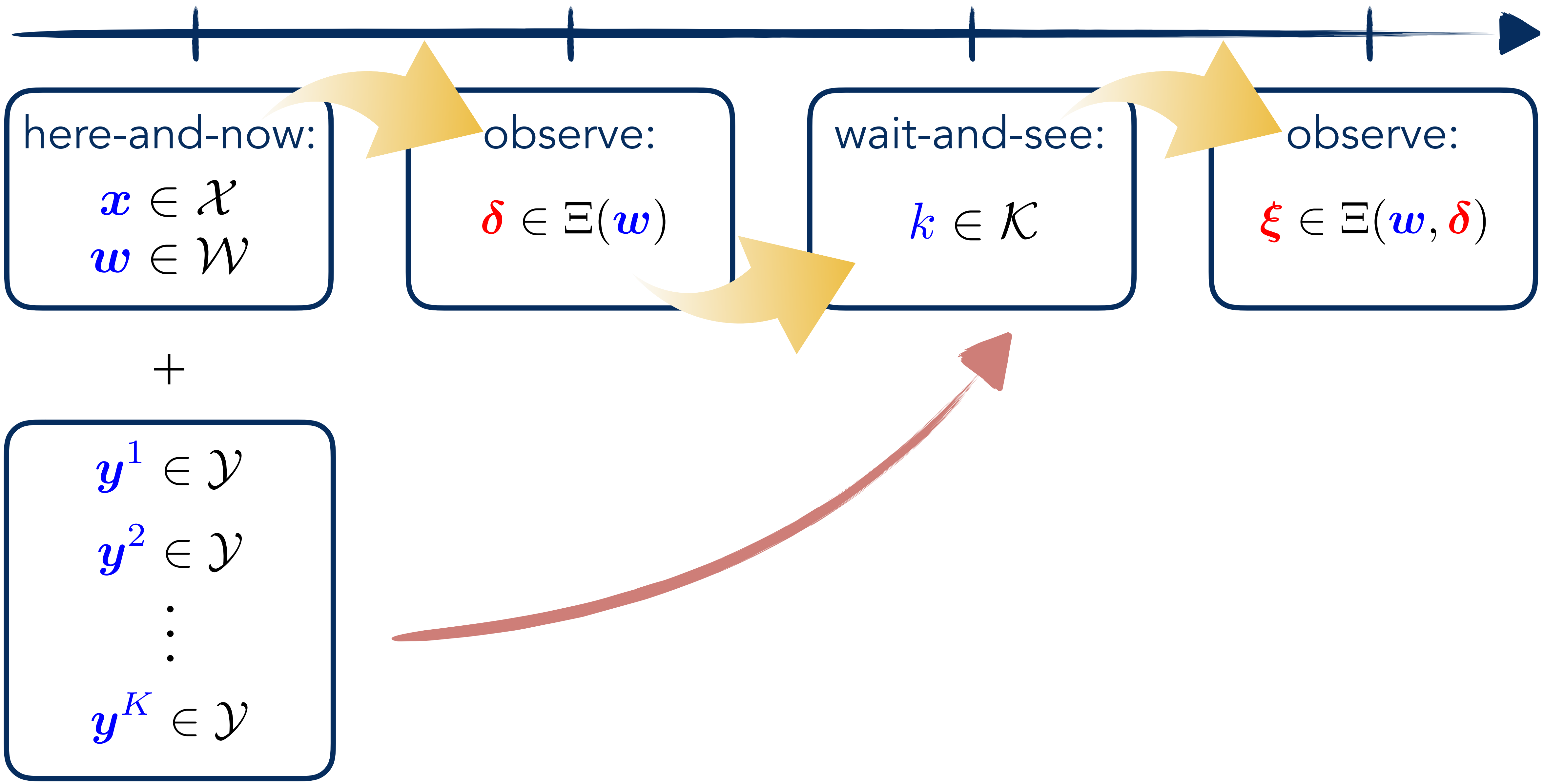
RO with DDID: K-Adaptability



[1] Exogenous uncertainty: Hanasusanto et. al (2015), Caramanis and Bertsimas (2010)

RO with DDID: K-Adaptability

here-and-now:

$\boldsymbol{x} \in \mathcal{X}$

$\boldsymbol{w} \in \mathcal{W}$

$\boldsymbol{y}^k \in \mathcal{Y},\ k \in \mathcal{K}$

observe:

$\boldsymbol{\delta} \in \Xi(\boldsymbol{w})$

wait-and-see:

$k \in \mathcal{K}$

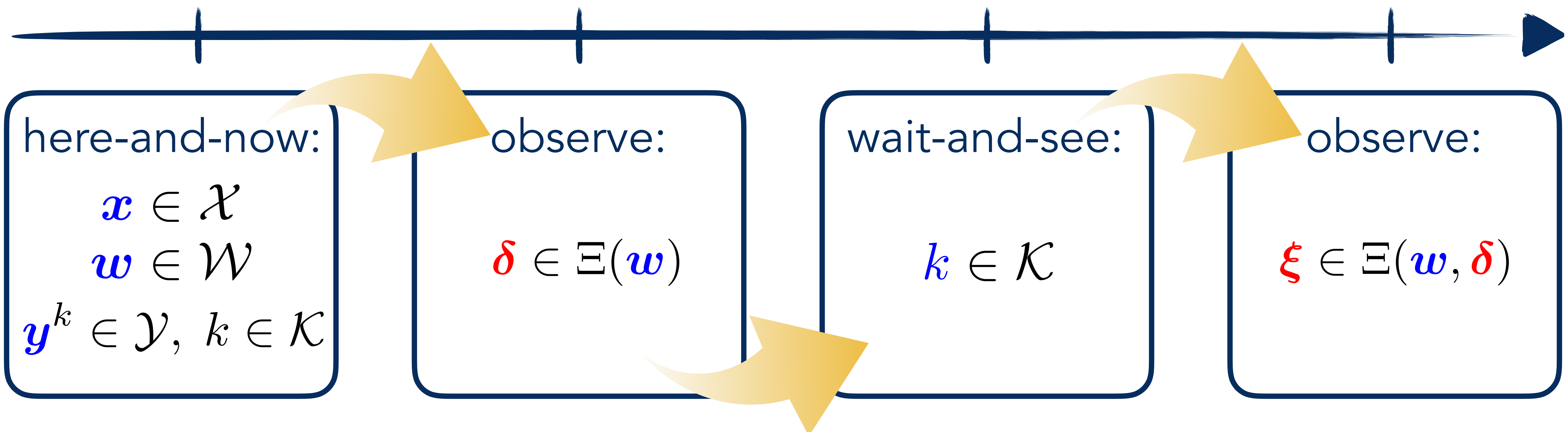
observe:

$\boldsymbol{\xi} \in \Xi(\boldsymbol{w}, \boldsymbol{\delta})$

K-Adaptability Problem

$$\begin{array}{lll} \underset{\substack{\boldsymbol{x}\in\mathcal{X},\ \boldsymbol{w}\in\mathcal{W} \\ \{\boldsymbol{y}^k\in\mathcal{Y}\}_{k\in\mathcal{K}}}}{\text{minimize}} & \underset{\boldsymbol{\delta}\in\Xi(\boldsymbol{w})}{\max}\ \underset{k\in\mathcal{K}}{\min}\ \underset{\boldsymbol{\xi}\in\Xi(\boldsymbol{w},\boldsymbol{\delta})}{\max} & \boldsymbol{\xi}^\top \boldsymbol{C}\boldsymbol{x} + \boldsymbol{\xi}^\top \boldsymbol{D}\boldsymbol{w} + \boldsymbol{\xi}^\top \boldsymbol{Q}\boldsymbol{y}^k \\ & \text{s.t.} & \boldsymbol{T}\boldsymbol{x} + \boldsymbol{V}\boldsymbol{w} + \boldsymbol{W}\boldsymbol{y}^k \leq \boldsymbol{h}(\boldsymbol{\xi}) \quad \forall \boldsymbol{\xi} \in \Xi(\boldsymbol{w}, \boldsymbol{\delta}) \end{array}$$

RO with DDID: K-Adaptability



K-Adaptability Problem

$$\begin{array}{ll} \underset{\substack{\boldsymbol{x}\in\mathcal{X},\ \boldsymbol{w}\in\mathcal{W} \\ \{\boldsymbol{y}^k\in\mathcal{Y}\}_{k\in\mathcal{K}}}}{\text{minimize}} \quad \max_{\boldsymbol{\delta}\in\Xi(\boldsymbol{w})} \quad \min_{k\in\mathcal{K}} \quad \max_{\boldsymbol{\xi}\in\Xi(\boldsymbol{w},\boldsymbol{\delta})} & \boldsymbol{\xi}^\top \boldsymbol{C}\boldsymbol{x} + \boldsymbol{\xi}^\top \boldsymbol{D}\boldsymbol{w} + \boldsymbol{\xi}^\top \boldsymbol{Q}\boldsymbol{y}^k \\ \qquad\qquad\qquad \text{s.t.} & \boldsymbol{T}\boldsymbol{x} + \boldsymbol{V}\boldsymbol{w} + \boldsymbol{W}\boldsymbol{y}^k \leq \boldsymbol{h}(\boldsymbol{\xi}) \quad \forall \boldsymbol{\xi} \in \Xi(\boldsymbol{w}, \boldsymbol{\delta}) \end{array}$$

Tractability?

Objective Uncertainty

K-Adaptability: MILP Reformulation

$$
\begin{array}{ll}
\text{minimize} & \boldsymbol{b}^\top \boldsymbol{\beta} + \sum_{k \in \mathcal{K}} \boldsymbol{b}^\top \boldsymbol{\beta}^k \\
\text{subject to} & \boldsymbol{x} \in \mathcal{X}, \ \boldsymbol{w} \in \mathcal{W}, \ \{\boldsymbol{y}^k\}_{k \in \mathcal{K}} \\
& \boldsymbol{\alpha} \in \mathbb{R}^K_+, \ \boldsymbol{\beta} \in \mathbb{R}^R_+, \ \boldsymbol{\beta}^k \in \mathbb{R}^R_+, \ \boldsymbol{\gamma}^k \in \mathbb{R}^{N_\xi}, \ k \in \mathcal{K} \\
& \mathbf{e}^\top \boldsymbol{\alpha} = 1 \\
& \boldsymbol{A}^\top \boldsymbol{\beta}^k + \boldsymbol{w} \circ \boldsymbol{\gamma}^k = \boldsymbol{\alpha}_k \left(\boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}\boldsymbol{w} + \boldsymbol{Q}\boldsymbol{y}^k \right) \quad \forall k \in \mathcal{K} \\
& \boldsymbol{A}^\top \boldsymbol{\beta} = \sum_{k \in \mathcal{K}} \boldsymbol{w} \circ \boldsymbol{\gamma}^k \\
& \boldsymbol{T}\boldsymbol{x} + \boldsymbol{V}\boldsymbol{w} + \boldsymbol{W}\boldsymbol{y}^k \leq \boldsymbol{h}
\end{array}
$$

The size of this problem is
polynomial in the size of the input

Two-Stage RO with DDID

K-Adaptability Problem

$$\begin{array}{ll} \underset{\substack{\boldsymbol{x}\in\mathcal{X},\, \boldsymbol{w}\in\mathcal{W} \\ \{\boldsymbol{y}^k\in\mathcal{Y}\}_{k\in\mathcal{K}}}}{\text{minimize}} & \max_{\boldsymbol{\delta}\in\Xi(\boldsymbol{w})} \; \min_{k\in\mathcal{K}} \; \max_{\boldsymbol{\xi}\in\Xi(\boldsymbol{w},\boldsymbol{\delta})} \; \boldsymbol{\xi}^\top \boldsymbol{C}\boldsymbol{x} + \boldsymbol{\xi}^\top \boldsymbol{D}\boldsymbol{w} + \boldsymbol{\xi}^\top \boldsymbol{Q}\boldsymbol{y}^k \\ & \text{s.t.} \quad \boldsymbol{T}\boldsymbol{x} + \boldsymbol{V}\boldsymbol{w} + \boldsymbol{W}\boldsymbol{y}^k \leq \boldsymbol{h}(\boldsymbol{\xi}) \quad \forall \boldsymbol{\xi} \in \Xi(\boldsymbol{w},\boldsymbol{\delta}) \end{array}$$

Objective Uncertainty

- Equivalent reformulation
- Polynomial MILP for fixed K
- Polynomial in K

Constraint Uncertainty

- Approximate reformulation
- Exponential in K

Two-Stage RO with DDID

K-Adaptability Problem

$$\begin{array}{ll} \underset{\substack{\boldsymbol{x}\in\mathcal{X},\ \boldsymbol{w}\in\mathcal{W} \\ \{\boldsymbol{y}^k\in\mathcal{Y}\}_{k\in\mathcal{K}}}}{\text{minimize}} & \max_{\boldsymbol{\delta}\in\Xi(\boldsymbol{w})} \ \min_{k\in\mathcal{K}} \ \max_{\boldsymbol{\xi}\in\Xi(\boldsymbol{w},\boldsymbol{\delta})} \ \boldsymbol{\xi}^\top \boldsymbol{C}\boldsymbol{x} + \boldsymbol{\xi}^\top \boldsymbol{D}\boldsymbol{w} + \boldsymbol{\xi}^\top \boldsymbol{Q}\boldsymbol{y}^k \\ & \text{s.t.} \quad \boldsymbol{T}\boldsymbol{x} + \boldsymbol{V}\boldsymbol{w} + \boldsymbol{W}\boldsymbol{y}^k \leq \boldsymbol{h}(\boldsymbol{\xi}) \quad \forall \boldsymbol{\xi} \in \Xi(\boldsymbol{w},\boldsymbol{\delta}) \end{array}$$

Objective Uncertainty

- Equivalent reformulation
- Polynomial MILP for fixed K
- Polynomial in K

Constraint Uncertainty

- Approximate reformulation
- Exponential in K

Generalizes K-adaptability to DDID

Two-Stage RO with DDID

K-Adaptability Problem

$$\begin{array}{ll} \underset{\substack{\boldsymbol{x}\in\mathcal{X},\ \boldsymbol{w}\in\mathcal{W} \\ \{\boldsymbol{y}^k\in\mathcal{Y}\}_{k\in\mathcal{K}}}}{\text{minimize}} & \max_{\boldsymbol{\delta}\in\Xi(\boldsymbol{w})} \ \min_{k\in\mathcal{K}} \ \max_{\boldsymbol{\xi}\in\Xi(\boldsymbol{w},\boldsymbol{\delta})} \left\{ \max_{i\in\mathcal{I}} \ \boldsymbol{\xi}^\top \boldsymbol{C}^i \boldsymbol{x} + \boldsymbol{\xi}^\top \boldsymbol{D}^i \boldsymbol{w} + \boldsymbol{\xi}^\top \boldsymbol{Q}^i \boldsymbol{y}^k \right\} \\ & \text{s.t.} \quad \boldsymbol{T}\boldsymbol{x} + \boldsymbol{V}\boldsymbol{w} + \boldsymbol{W}\boldsymbol{y}^k \leq \boldsymbol{h} \end{array}$$

Piecewise Linear Convex Objective

- Equivalent reformulation
- Exponential in K
- Efficient column-and-constraint generation

Two-Stage RO with DDID

K-Adaptability Problem

$$\begin{aligned} \underset{\substack{\boldsymbol{x}\in\mathcal{X},\ \boldsymbol{w}\in\mathcal{W} \\ \{\boldsymbol{y}^k\in\mathcal{Y}\}_{k\in\mathcal{K}}}}{\text{minimize}} \quad \max_{\boldsymbol{\delta}\in\Xi(\boldsymbol{w})} \quad \min_{k\in\mathcal{K}} \quad \max_{\boldsymbol{\xi}\in\Xi(\boldsymbol{w},\boldsymbol{\delta})} & \left\{ \max_{i\in\mathcal{I}} \ \boldsymbol{\xi}^\top \boldsymbol{C}^i \boldsymbol{x} + \boldsymbol{\xi}^\top \boldsymbol{D}^i \boldsymbol{w} + \boldsymbol{\xi}^\top \boldsymbol{Q}^i \boldsymbol{y}^k \right\} \\ \text{s.t.} \quad & \boldsymbol{T}\boldsymbol{x} + \boldsymbol{V}\boldsymbol{w} + \boldsymbol{W}\boldsymbol{y}^k \leq \boldsymbol{h} \end{aligned}$$

Piecewise Linear Convex Objective

- Equivalent reformulation
- Exponential in K
- Efficient column-and-constraint generation

Generalizes K-adaptability to nonlinear objective

Max-Min Utility

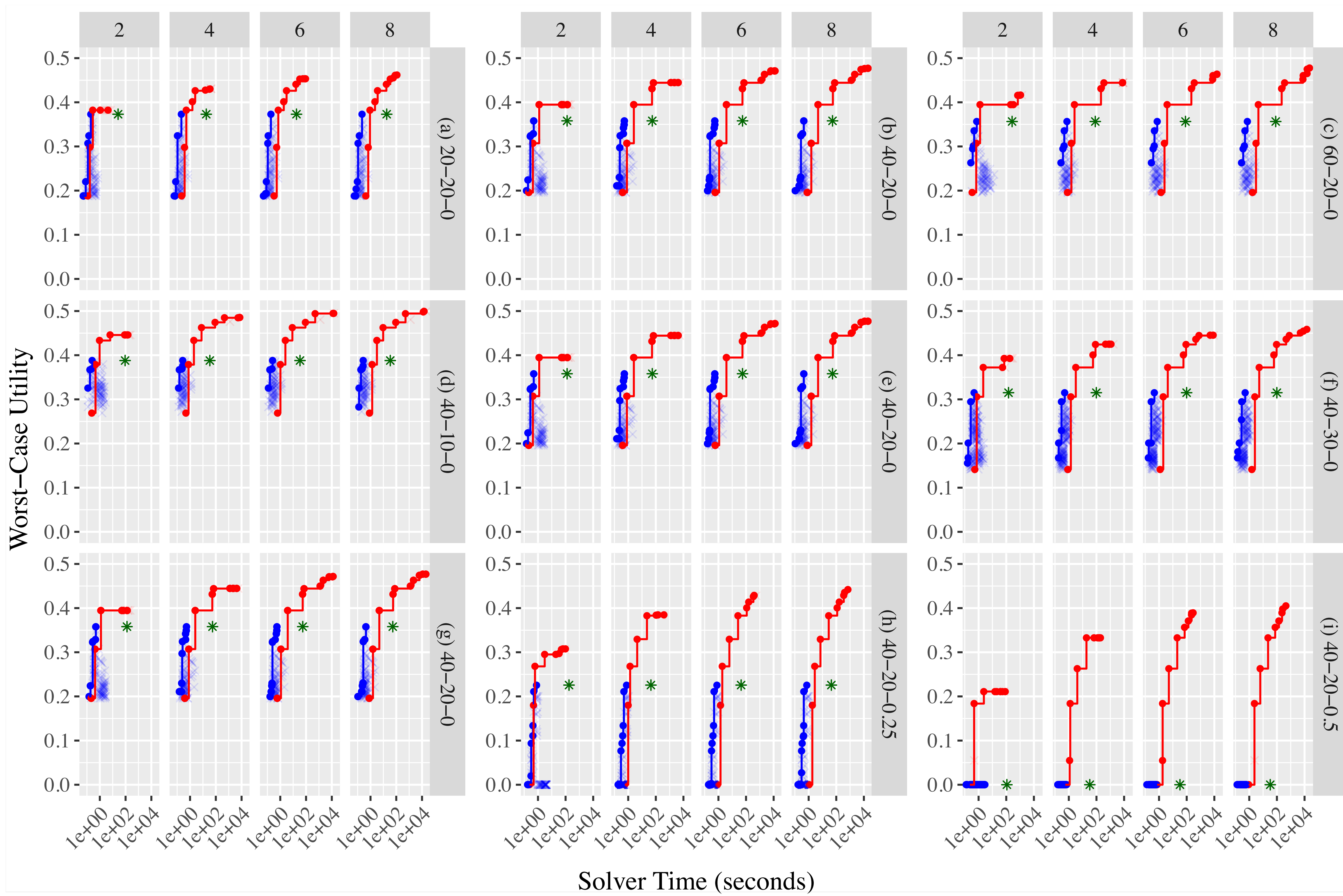
Problem Formulation

$$\begin{array}{ll} \text{maximize} & \min_{\overline{\boldsymbol{\xi}} \in \Xi} \ \max_{\boldsymbol{y} \in \mathcal{Y}} \left\{ \min_{\boldsymbol{\xi} \in \Xi(\boldsymbol{w}, \overline{\boldsymbol{\xi}})} \ \boldsymbol{\xi}^\top \boldsymbol{y} \right\} \\ \text{subject to} & \boldsymbol{w} \in \mathcal{W} \end{array}$$

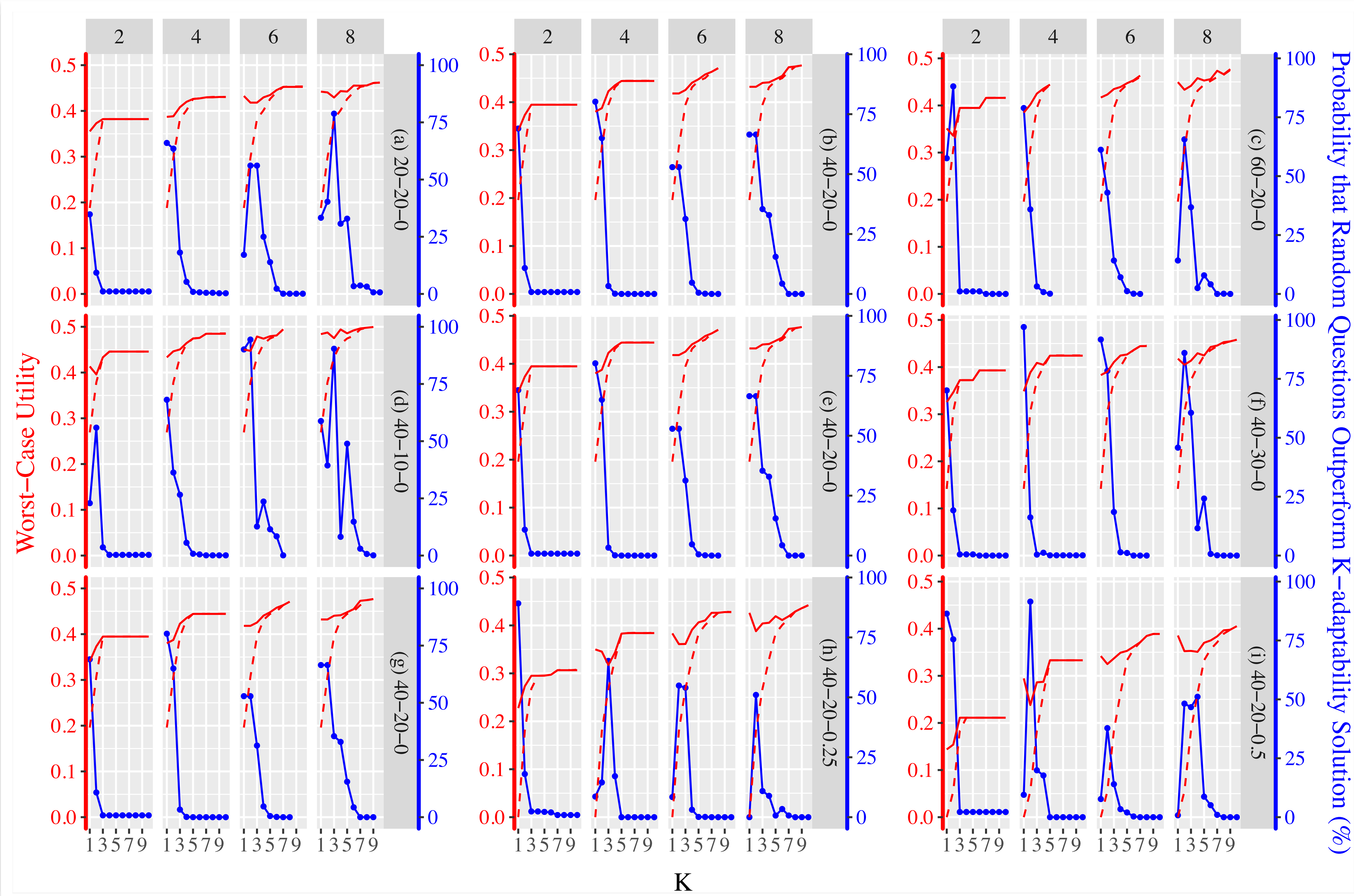
$$\mathcal{W} := \left\{ \boldsymbol{w} \in \{0,1\}^I \ : \ \sum_{i \in \mathcal{I}} \boldsymbol{w}_i = Q \right\}$$

$$\mathcal{Y} := \left\{ \boldsymbol{y} \in \{0,1\}^I \ : \ \sum_{i \in \mathcal{I}} \boldsymbol{y}_i = 1 \right\}$$

Max-Min Utility Synthetic Data



Max-Min Utility Synthetic Data



Min-Max Regret

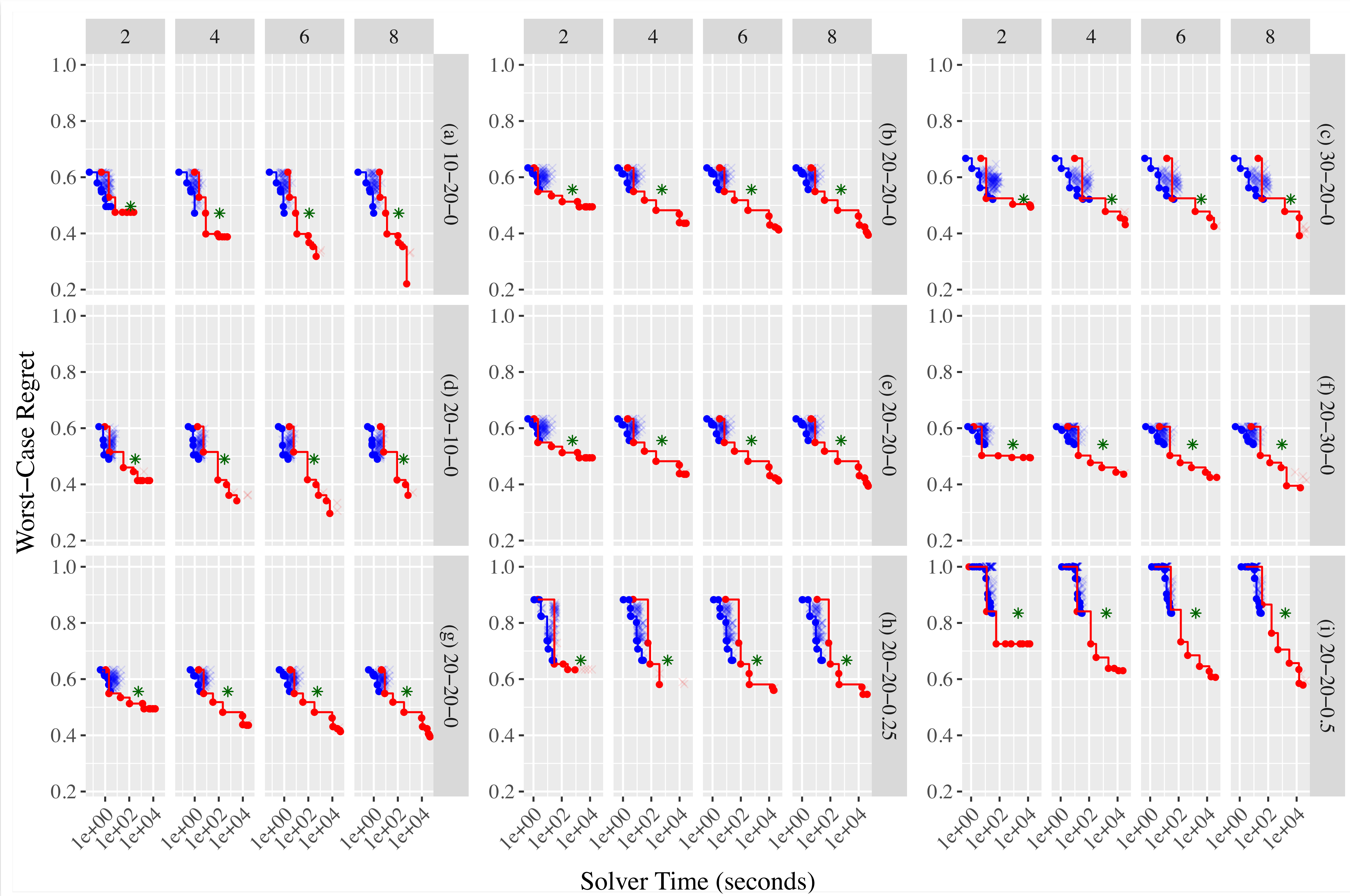
Problem Formulation

$$\text{minimize}_{\boldsymbol{w}\in\mathcal{W}} \quad \max_{\overline{\boldsymbol{\xi}}\in\Xi} \; \min_{\boldsymbol{y}\in\mathcal{Y}} \; \max_{\boldsymbol{\xi}\in\Xi(\boldsymbol{w},\overline{\boldsymbol{\xi}})} \left\{ \max_{i\in\mathcal{I}} \; \boldsymbol{\xi}_i - \boldsymbol{\xi}^\top \boldsymbol{y} \right\}$$

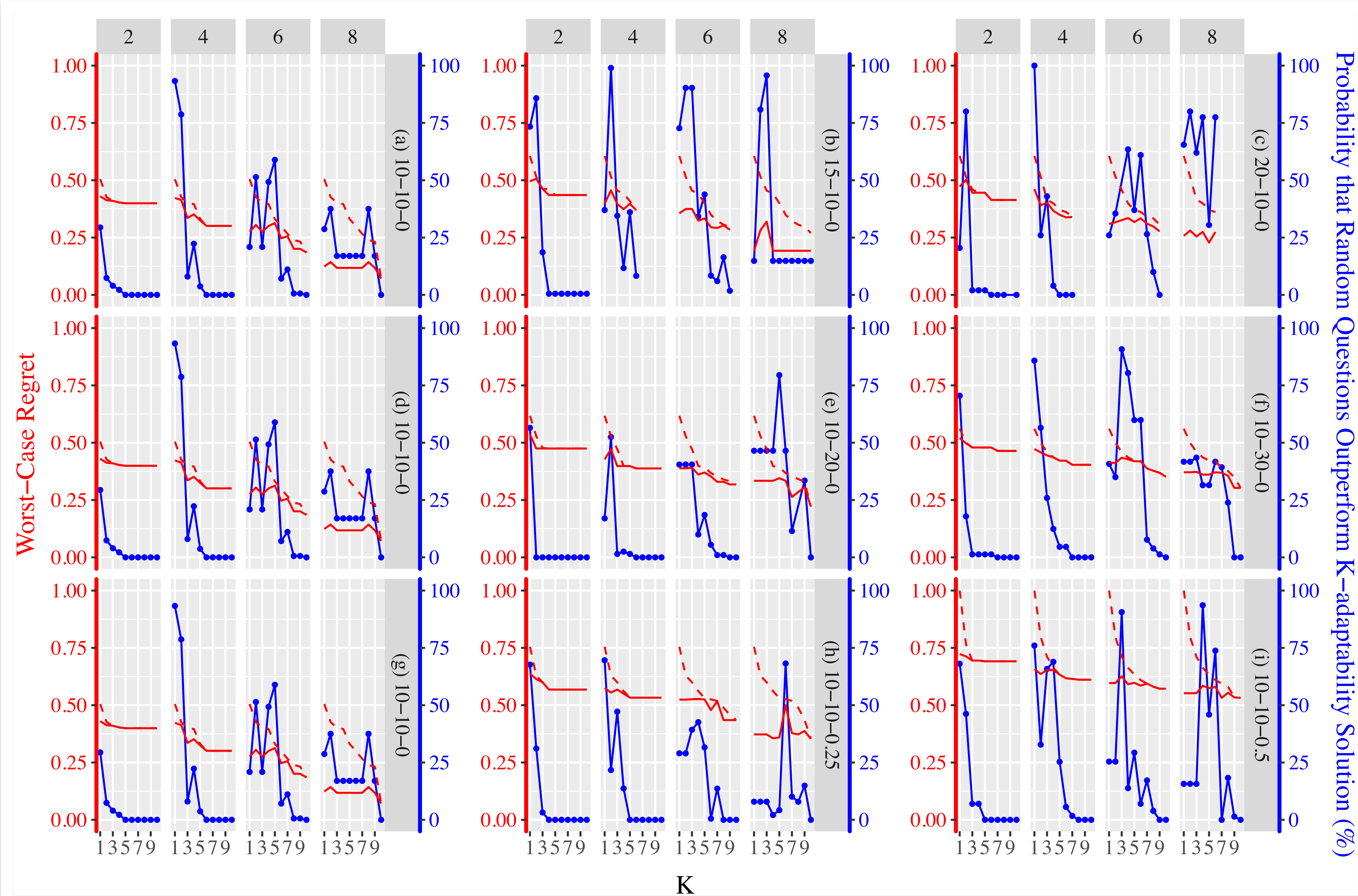
$$\mathcal{W} := \left\{ \boldsymbol{w} \in \{0,1\}^I \; : \; \sum_{i\in\mathcal{I}} \boldsymbol{w}_i = Q \right\}$$

$$\mathcal{Y} := \left\{ \boldsymbol{y} \in \{0,1\}^I \; : \; \sum_{i\in\mathcal{I}} \boldsymbol{y}_i = 1 \right\}$$

Min-Max Regret Synthetic Data

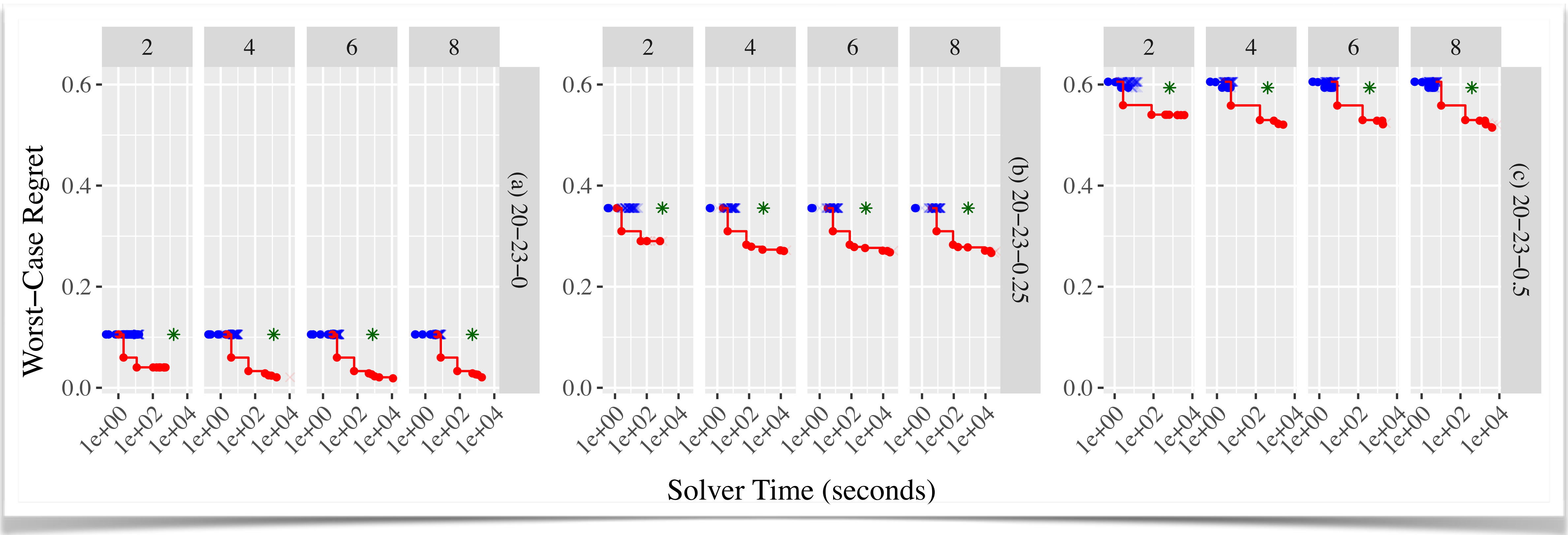


Min-Max Regret Synthetic Data



Min-Max Regret LAHSA Data

- **Simulated the outcomes of 20 policies, including:**
  - Including current policy, random allocation, FCFS
  - Used real data from HMIS
  - 23 features that characterize fairness, efficiency, interpretability



Min-Max Regret LAHSA Data

- **Simulated the outcomes of 20 policies, including:**
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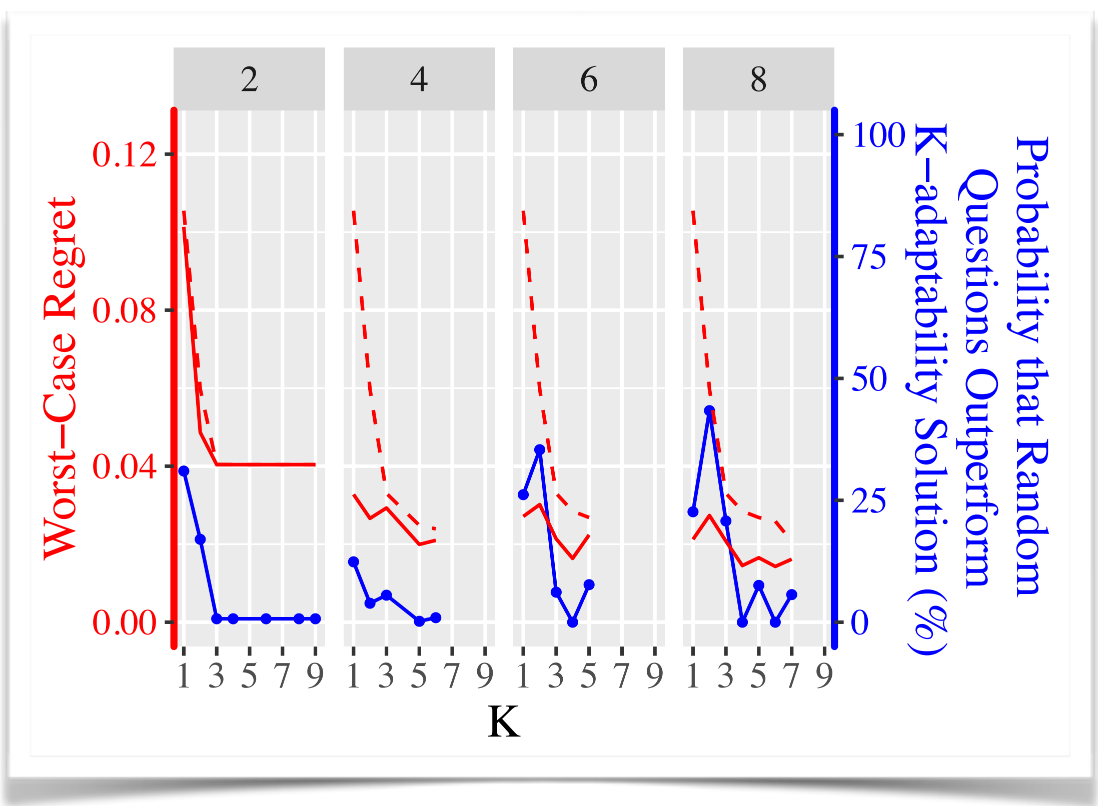
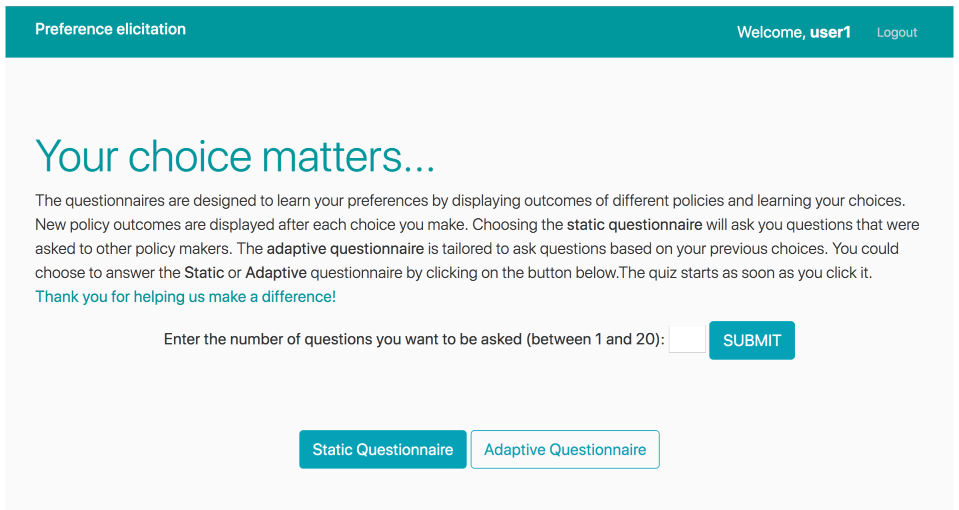
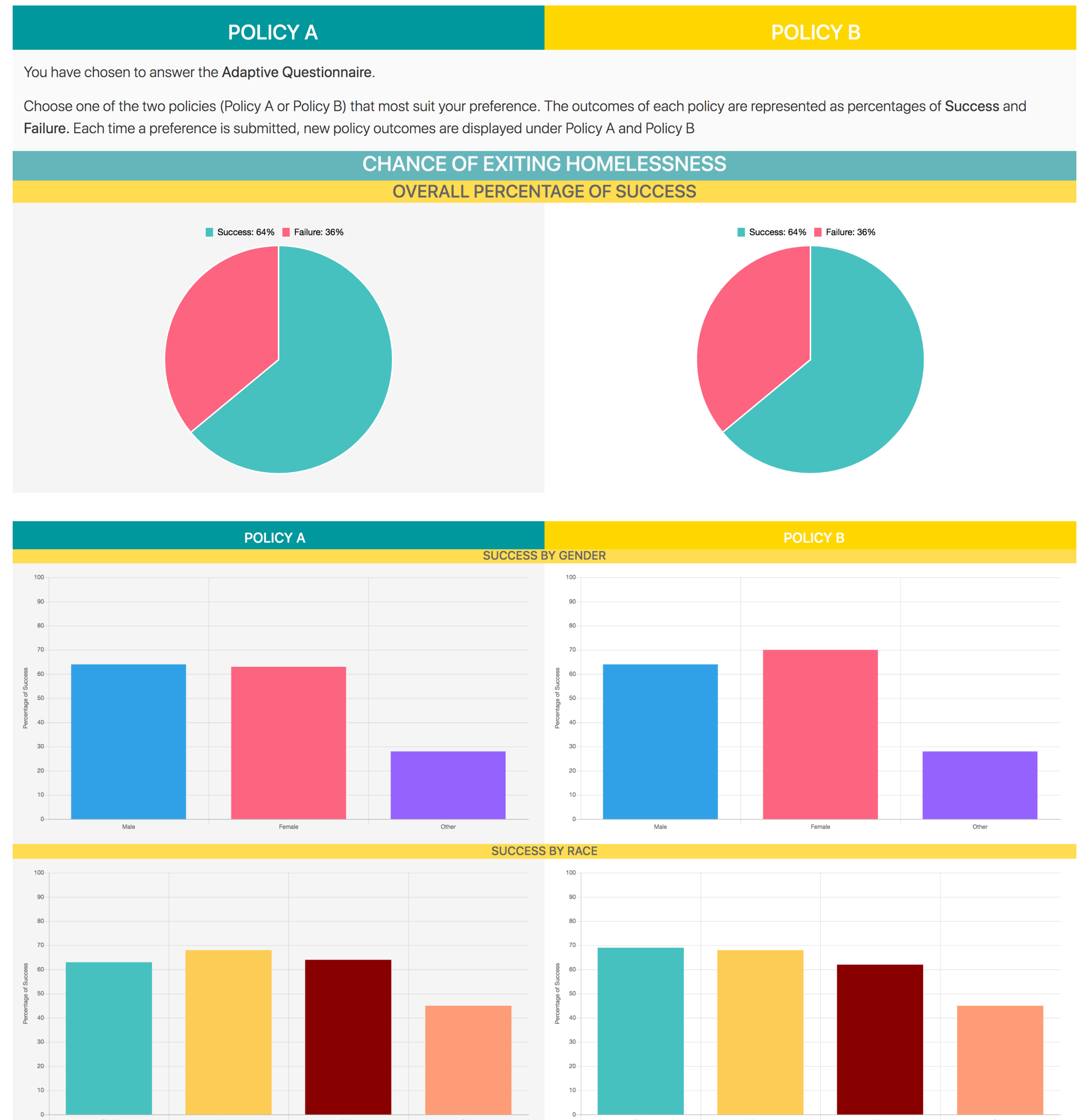


Towards Real-World Deployment





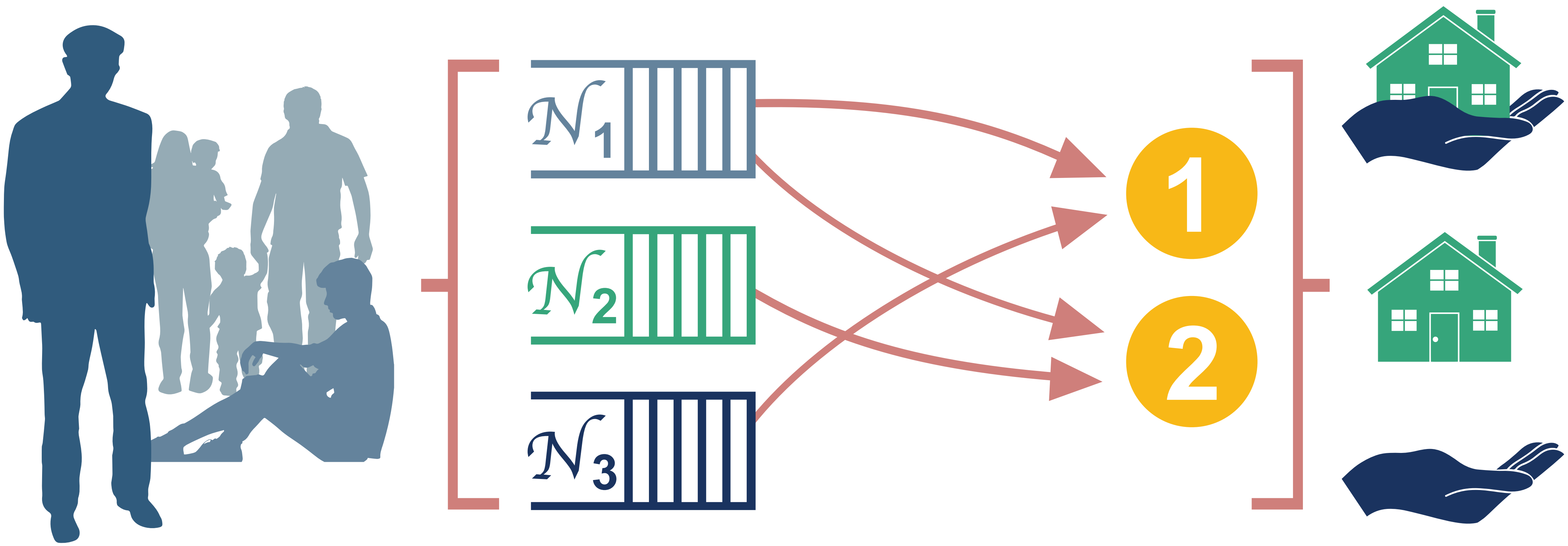
LOS ANGELES HOMELESS SERVICES AUTHORITY
LAHSA

First field tests forthcoming!

Outline

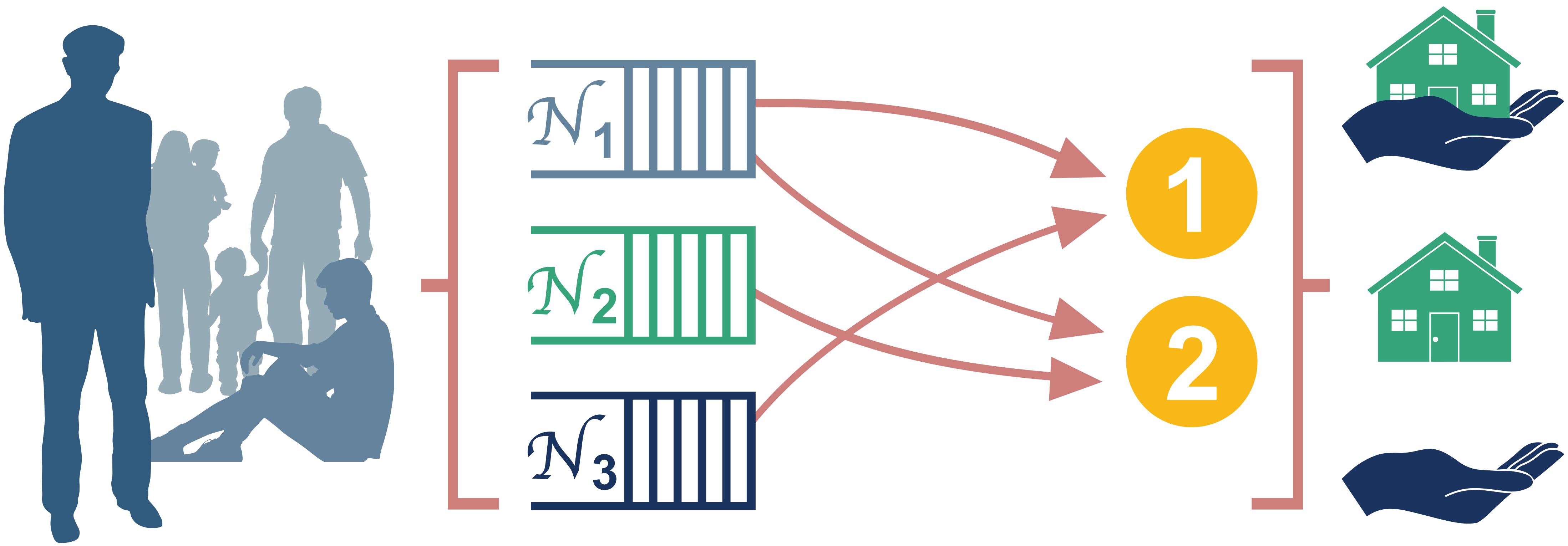
- ☑ Estimating Wait Times in Resource Allocation Systems
- ☐ Designing Policies for Allocating Scarce Resources
  - ☑ Preference Elicitation
  - ☐ Policy Optimization
- ☐ Optimizing "Gatekeeper Trainings" for Suicide Prevention

System Model



- Features of house: $\mathscr{F}_h \in \mathbb{R}^{n_\mathrm{h}}$
- Features of the youth: $\mathscr{G}_y \in \mathbb{R}^{n_\mathrm{y}}$
- Youth eligible for house if and only if: $\mathscr{G}_y \in \mathbb{M}(\mathscr{F}_h)$

System Model



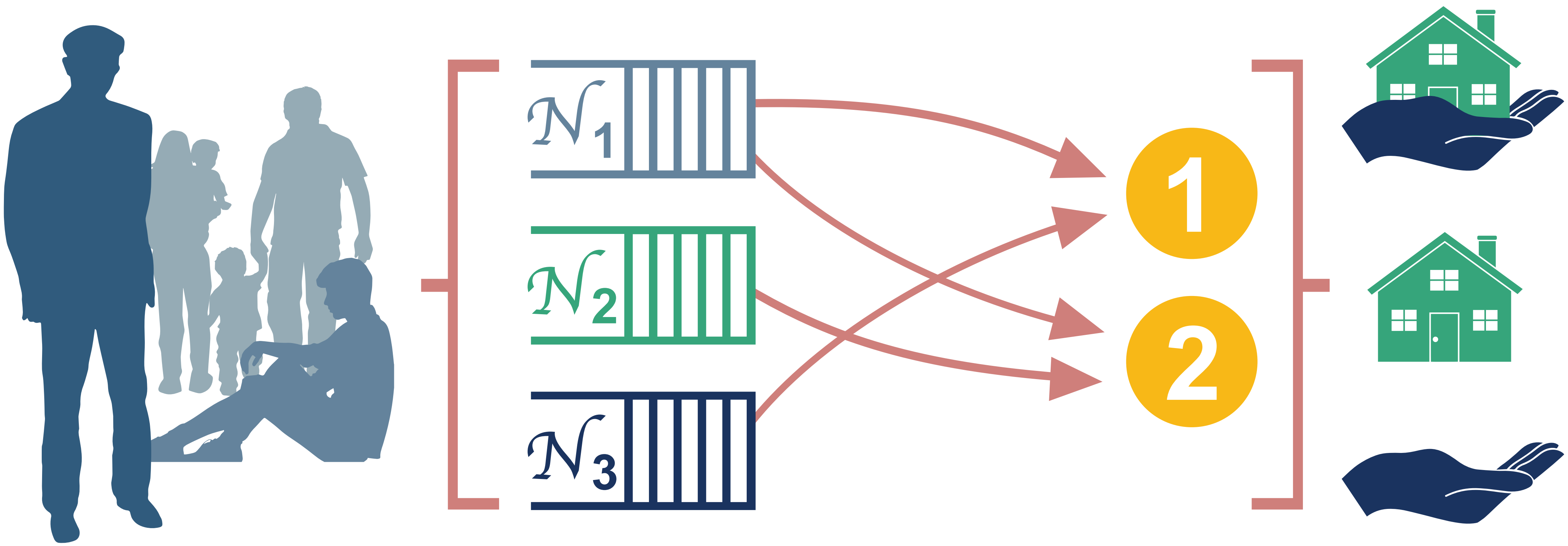
- Probability of successful outcome with house:

  $$p(\mathscr{G}_y, \mathscr{F}_h)$$

- Probability of successful outcome without house:

  $$\overline{p}(\mathscr{G}_y)$$

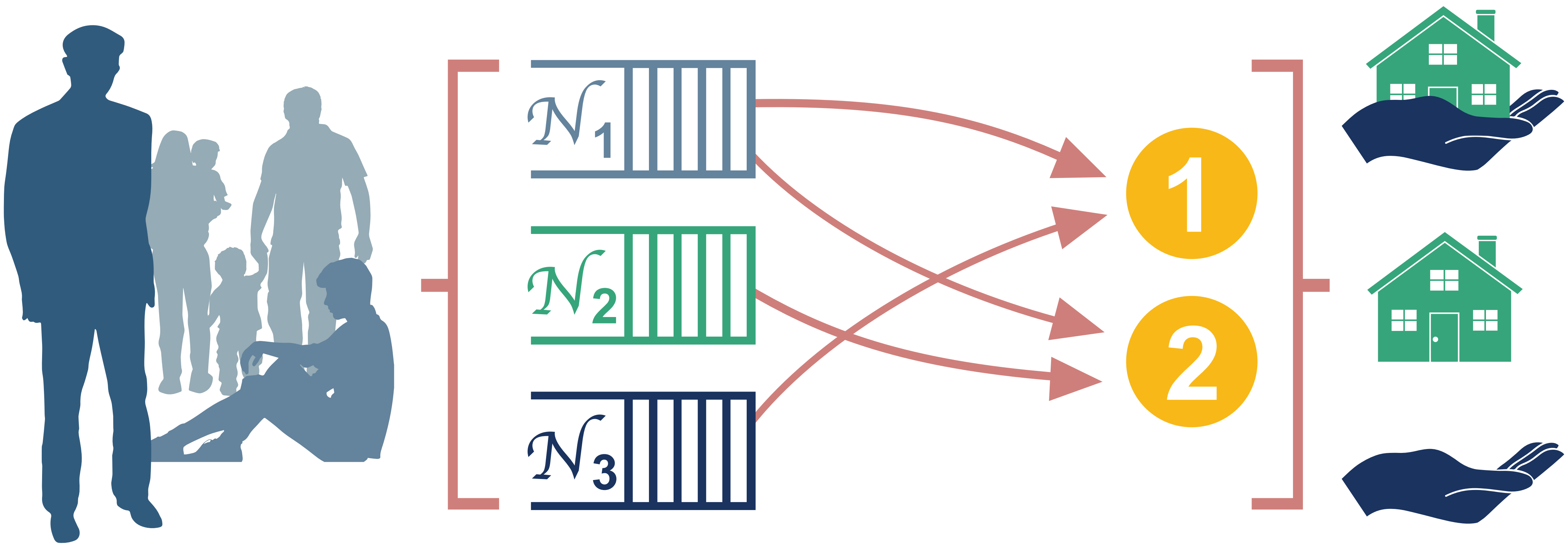
Parametric Scoring Policies



- Parameter vector: $\beta \in \mathbb{R}^n$
- Score for particular matching: $\pi_\beta(\mathscr{G}_y, \mathscr{F}_h)$
- Youth y has priority over youth y' if:

$$\pi_\beta(\mathscr{G}_y, \mathscr{F}_h) > \pi_\beta(\mathscr{G}_{y'}, \mathscr{F}_h)$$

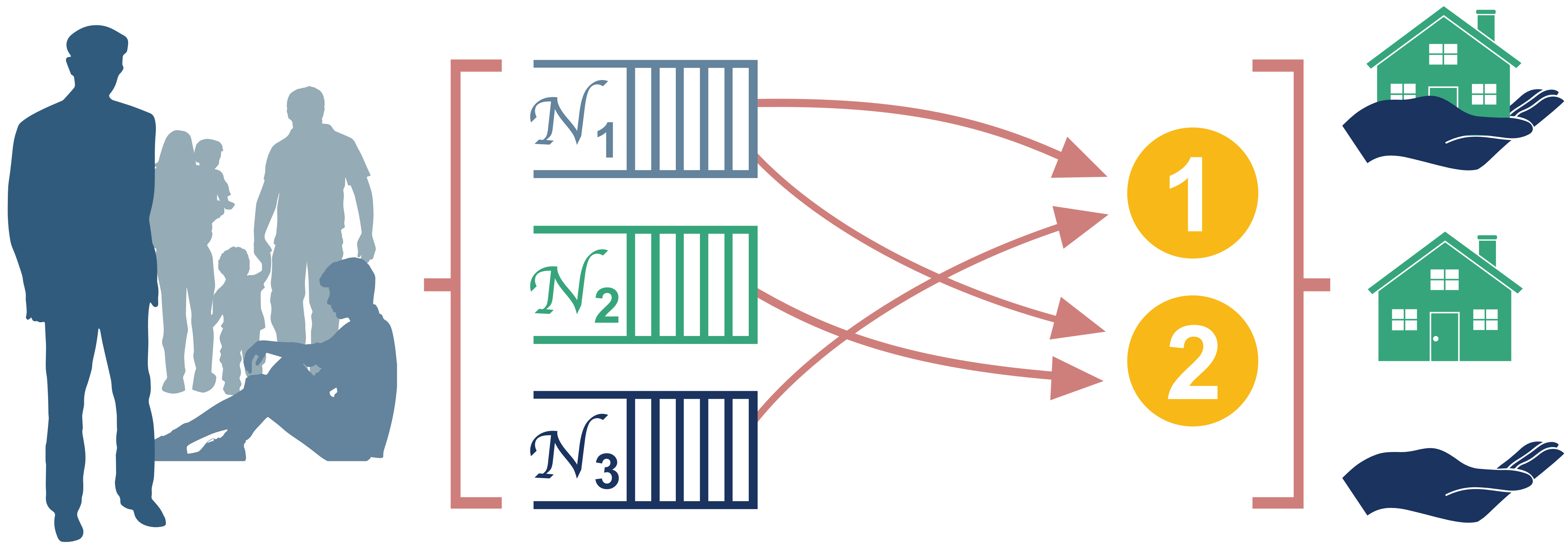
Parametric Scoring Policies



- Parameter vector: $\beta \in \mathbb{R}^n$
- Score for particular matching: $\pi_\beta(\mathscr{G}_y, \mathscr{F}_h)$ ← chosen by user
- Youth y has priority over youth y' if:

$$\pi_\beta(\mathscr{G}_y, \mathscr{F}_h) > \pi_\beta(\mathscr{G}_{y'}, \mathscr{F}_h)$$

Parametric Scoring Policies



- Parameter vector: $\beta \in \mathbb{R}^n$
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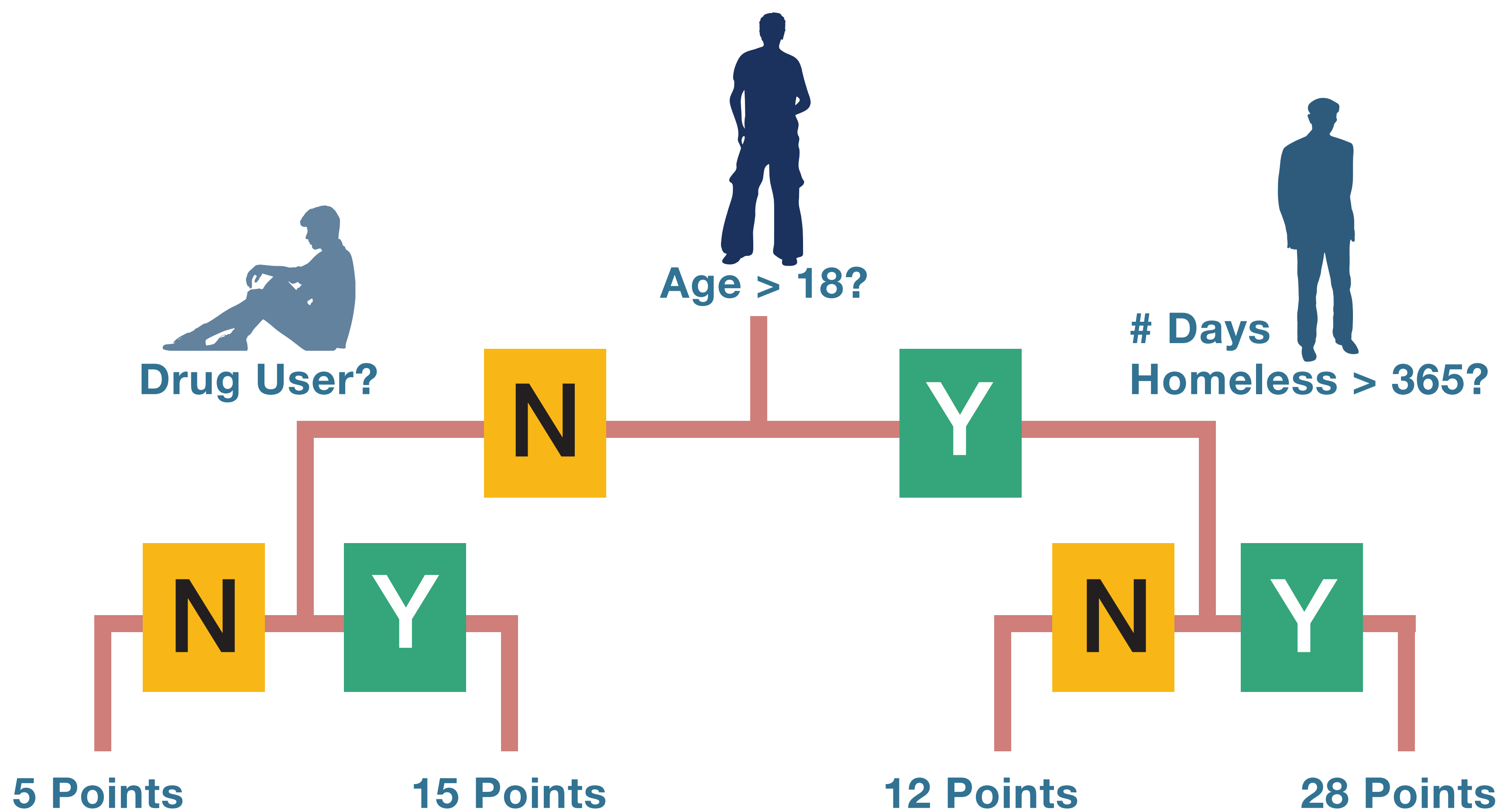
- Youth y has priority over youth y' if:

$$\pi_\beta(\mathscr{G}_y, \mathscr{F}_h) > \pi_\beta(\mathscr{G}_{y'}, \mathscr{F}_h)$$

chosen by user

optimize!

Interpretable Policies



- Linear policies
- Decision-tree based policies with linear leafing/branching

Fair Policies

Fair Policies

- Probability of successful outcome should be independent of one's race

Fair Policies

- Probability of successful outcome should be independent of one's race

- Probability of successful outcome should be independent of one's gender

Fair Policies

- Probability of successful outcome should be independent of one's race
- Probability of successful outcome should be independent of one's gender
- Probability of successful outcome should be independent of one's vulnerability score

Fair Policies

Freedom to Incorporate General Criteria!

- Probability of successful outcome should be independent of one's race
- Probability of successful outcome should be independent of one's gender
- Probability of successful outcome should be independent of one's vulnerability score

Data-Driven Optimization

maximize $\sum_{y\in\mathbb{Y}}\left[\sum_{h\in\mathbb{H}} p_{yh}x_{yh} + \overline{p}_y\left(1-\sum_{h\in\mathbb{H}} x_{yh}\right)\right]$

subject to $\pi_{yh} = \pi_\beta(g_y, f_h), \quad \forall y\in\mathbb{Y},\ h\in\mathbb{H}$

$\forall y \in \mathbb{Y},\ h\in\mathbb{H},$

$$x_{yh}=1 \Leftrightarrow \left\{\begin{array}{l} (y,h)\in\mathbb{C}, \quad \sum_{h'\neq h:\alpha_{h'}\leq\alpha_h} x_{yh'} = 0, \quad \text{and} \\ \forall y' : (y',h)\in\mathbb{C} \ \text{ and } \sum_{h':\alpha_{h'}\leq\alpha_h} x_{y'h'} = 0, \\ (\pi_{yh} > \pi_{y'h}) \text{ or } (\pi_{yh} = \pi_{y'h} \text{ and } \rho_y > \rho_{y'}) \end{array}\right\}$$

$\beta\in\mathbb{B},\ x\in\mathbb{F},\ x_{yh}\in\{0,1\}\ \forall y\in\mathbb{Y},\ h\in\mathbb{H}.$

Equivalent to MILP for Interpretable Class of Policies

Proposed Solution Approach

Step 1: Solve Relaxation to Original Problem

$$\begin{aligned}
\text{maximize} \quad & \sum_{y \in \mathbb{Y}} \left[\sum_{h \in \mathbb{H}} p_{yh} x_{yh} + \overline{p}_y \left(1 - \sum_{h \in \mathbb{H}} x_{yh} \right) \right] \\
\text{subject to} \quad & \sum_{h \in \mathbb{H}} x_{yh} \leq 1 \quad \forall y \in \mathbb{Y}, \quad \sum_{y \in \mathbb{Y}} x_{yh} \leq 1 \quad \forall h \in \mathbb{H} \\
& x_{yh} = 0 \quad \forall y \in \mathbb{Y},\ h \in \mathbb{H} \ : \ (y,h) \notin \mathbb{C} \\
& x \in \mathbb{F},\ x_{yh} \geq 0 \quad \forall y \in \mathbb{Y},\ h \in \mathbb{H}
\end{aligned}$$

- Matching augmented with fairness constraints

$$\mathbb{F} := \{x : Ax \leq b\}$$

Talluri and van Ryzin, MS 1998

Bertsimas and Trichakis, OR 2013

Proposed Solution Approach

Step 1: Solve Relaxation to Original Problem

Equivalent to:

$$
\begin{aligned}
\text{maximize} \quad & \sum_{y \in \mathbb{Y}} \left[\sum_{h \in \mathbb{H}} p_{yh} x_{yh} + \overline{p}_y \left(1 - \sum_{h \in \mathbb{H}} x_{yh} \right) \right] - \lambda^\top A x + \lambda^\top b \\
\text{subject to} \quad & \sum_{h \in \mathbb{H}} x_{yh} \leq 1 \quad \forall y \in \mathbb{Y}, \quad \sum_{y \in \mathbb{Y}} x_{yh} \leq 1 \quad \forall h \in \mathbb{H} \\
& x_{yh} = 0 \quad \forall y \in \mathbb{Y},\ h \in \mathbb{H} \ : \ (y,h) \notin \mathbb{C} \\
& x_{yh} \geq 0 \quad \forall y \in \mathbb{Y},\ h \in \mathbb{H}
\end{aligned}
$$

Define:

$$C_{yh} := p_{yh} - \overline{p}_y - (\lambda^\top A)_{(y,h)}$$

Talluri and van Ryzin, MS 1998

Bertsimas and Trichakis, OR 2013

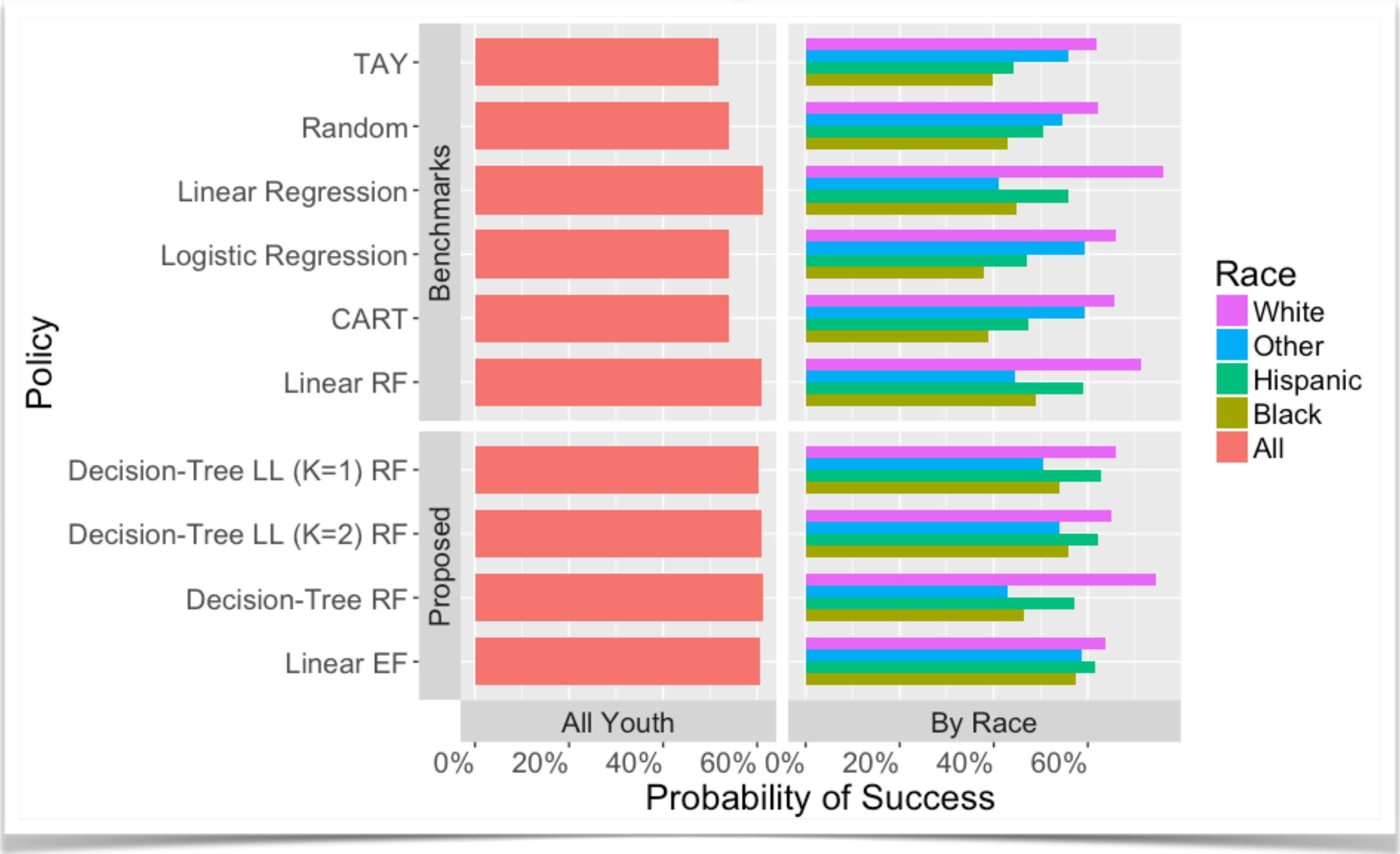
Approximate Solution Approach

Step 2: Learn Adjusted Success Probabilities

$$\begin{aligned} \text{minimize} \quad & \sum_{y \in \mathbb{Y}} \sum_{h \in \mathbb{H}} |C_{yh} - \pi_{yh}| \\ \text{subject to} \quad & \pi_{yh} = \pi_\beta(g_y, f_y) \end{aligned}$$

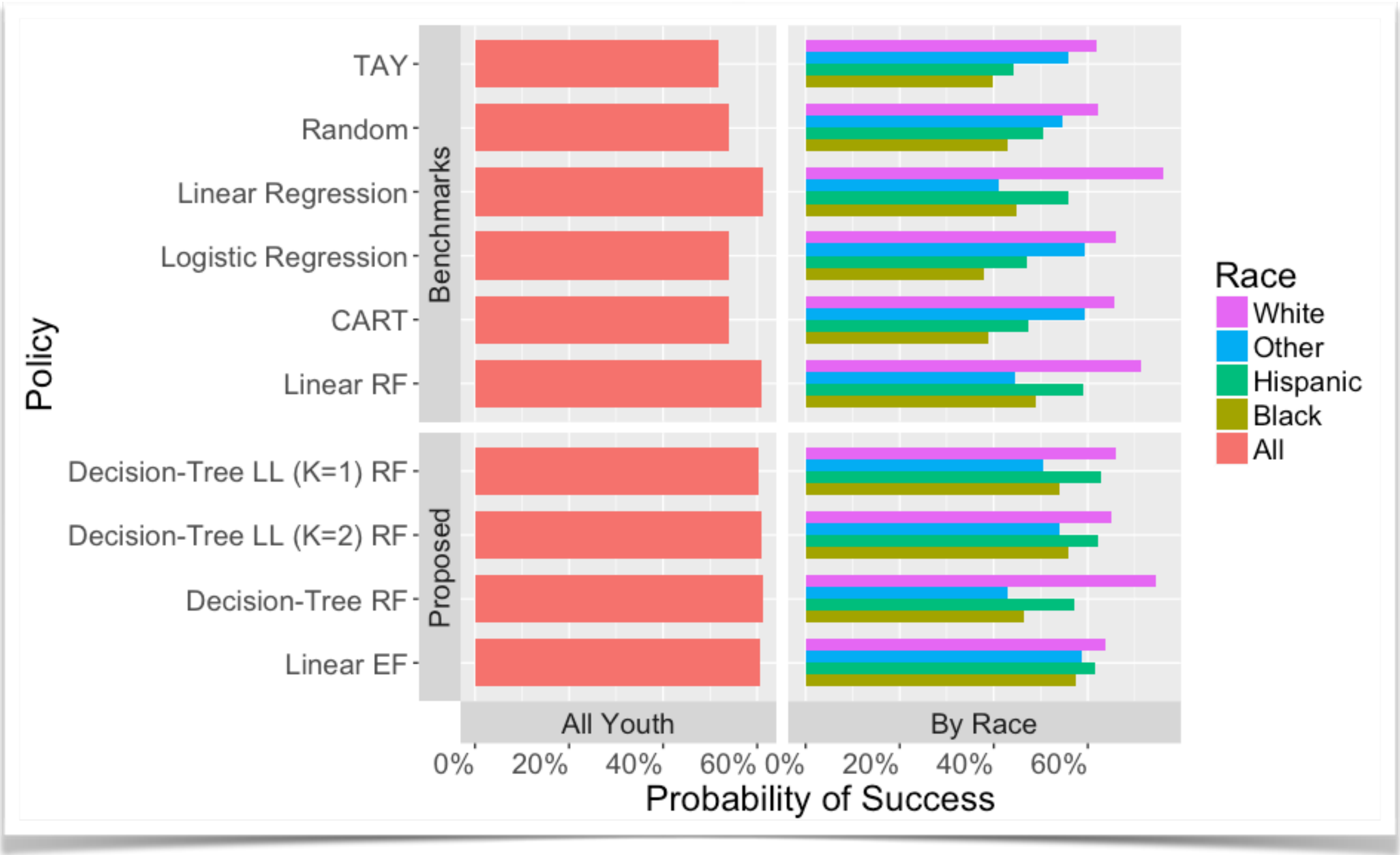
- An LP for linear policies
- A large scale MILP for decision-tree based policies
- Nice decomposable structure: solve using Bender's decomposition

Fairness Across Races



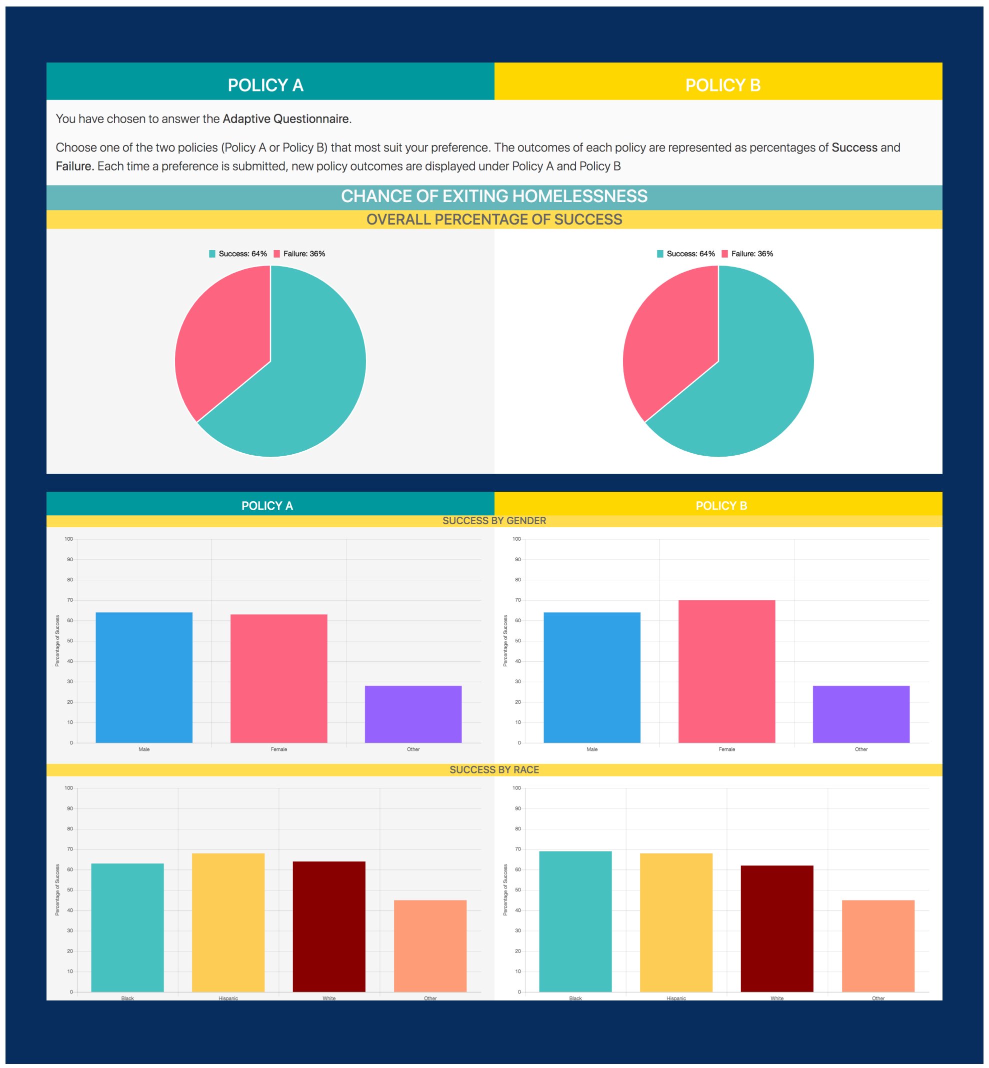
- Proposed policy mitigates 72 % of racial bias

Fairness Across Races



- Proposed policy increases efficiency by 16%

Towards Real World Deployment



LAHSA LOS ANGELES HOMELESS SERVICES AUTHORITY

Outline

- ☑ Estimating Wait Times in Resource Allocation Systems
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  - ☑ Preference Elicitation
  - ☑ Policy Optimization
- ☐ Optimizing "Gatekeeper Trainings" for Suicide Prevention

Partner

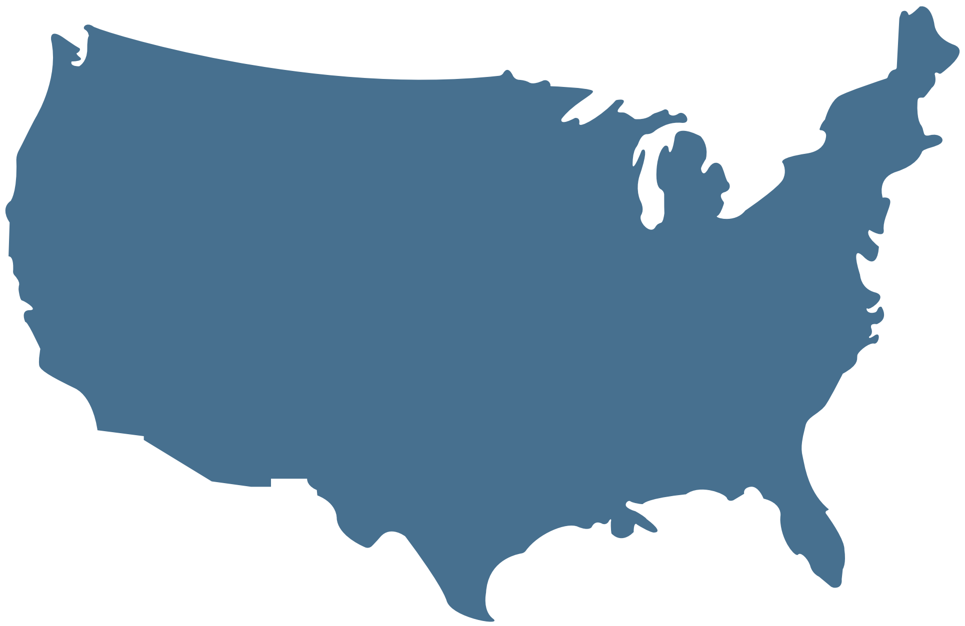
Anthony Fulginiti

Assistant Professor

DU School of Social Work

UNIVERSITY of DENVER

Alarming Rates of Suicide



- Suicide is the tenth leading cause of death overall
- Suicide is the fourth leading cause of death among ages 35-54
- Second leading cause of death among ages 10-34!
- In 2016, nearly 45,000 people died by suicide in the U.S.

National Health Care for the Homeless Council, Fact Sheet, 2018

A Personal Motivation

	UGs only	UGs + Gs combined	Not enrolled full-time, ages 18–22
seriously considered	6.6%–7.5%	7.1%–7.7%	9.0%
made a plan	2.2%–2.3%	2.3%	3.1%
attempted suicide	1.1%–1.2%	0.6%–1.2%	2.2%

- Suicide is the leading cause of death among college and university students!

Suicide Prevention Resource Center

"Gatekeeper" Training

- Most popular suicide prevention program
- Conducted among college students, military personnel, etc.
- Trains "helpers" to identify warning signs of suicide and how to respond

"Gatekeeper" Training

- Most popular suicide prevention program
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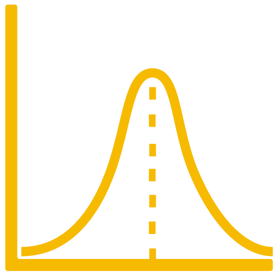
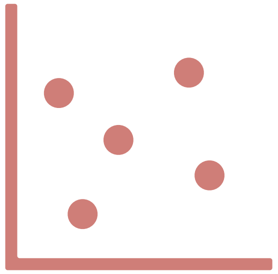
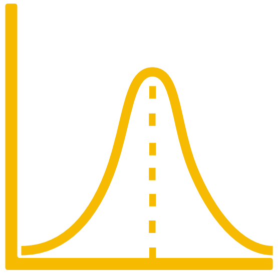
Can we leverage social network information?

Challenges

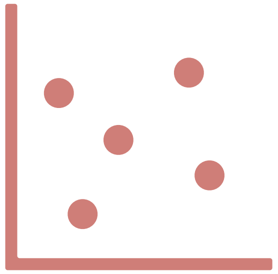




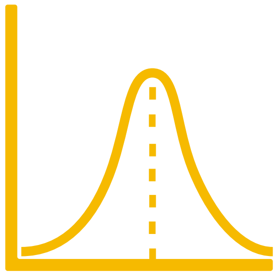
Challenges



- Uncertainty in availability and performance of students



Challenges



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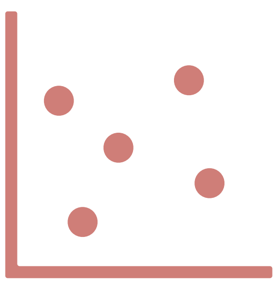
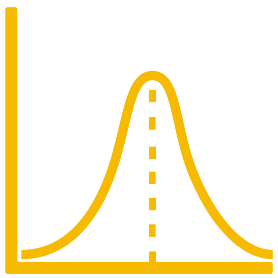


- In practice: limited data to inform node availability

Challenges

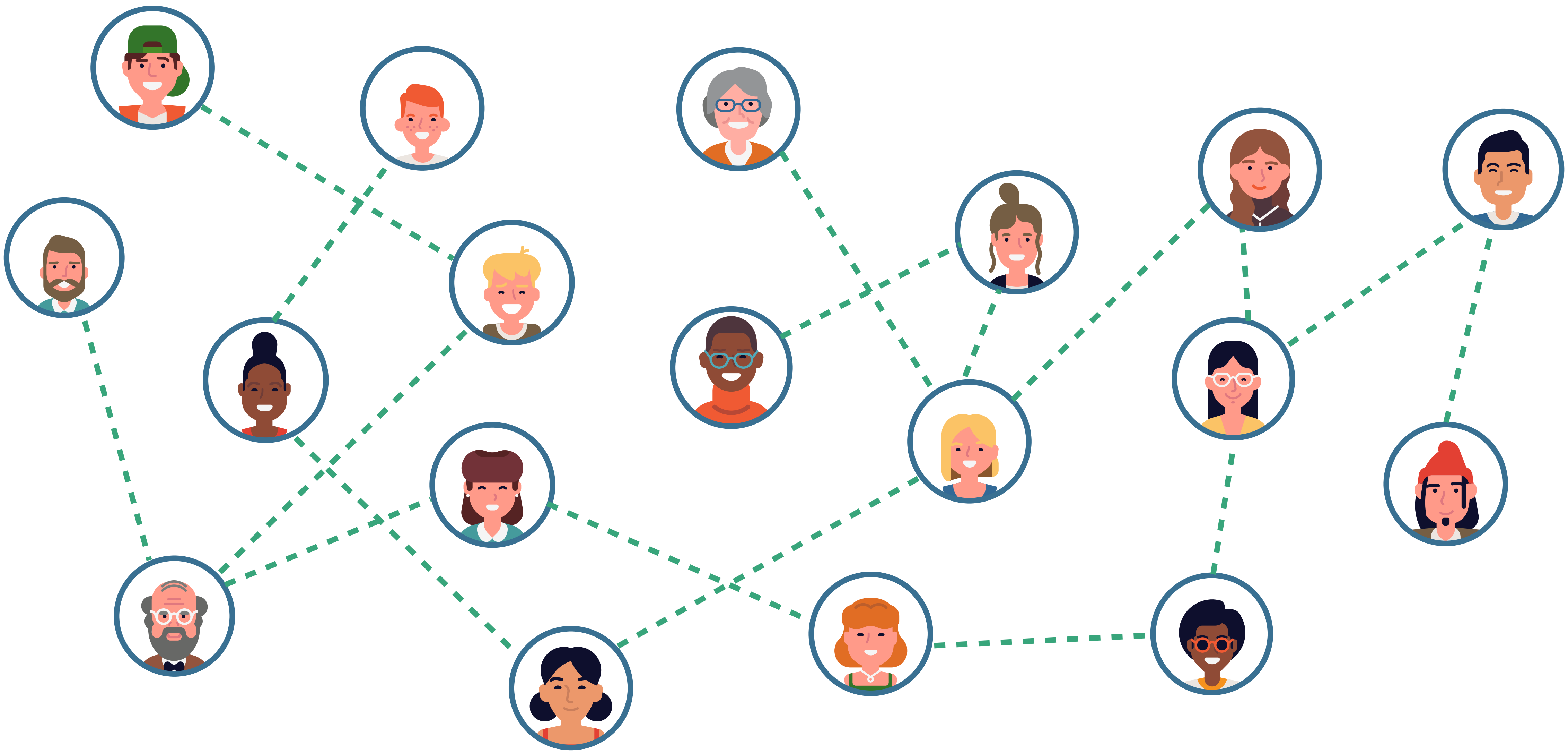


- Uncertainty in availability and performance of students

- In practice: limited data to inform node availability

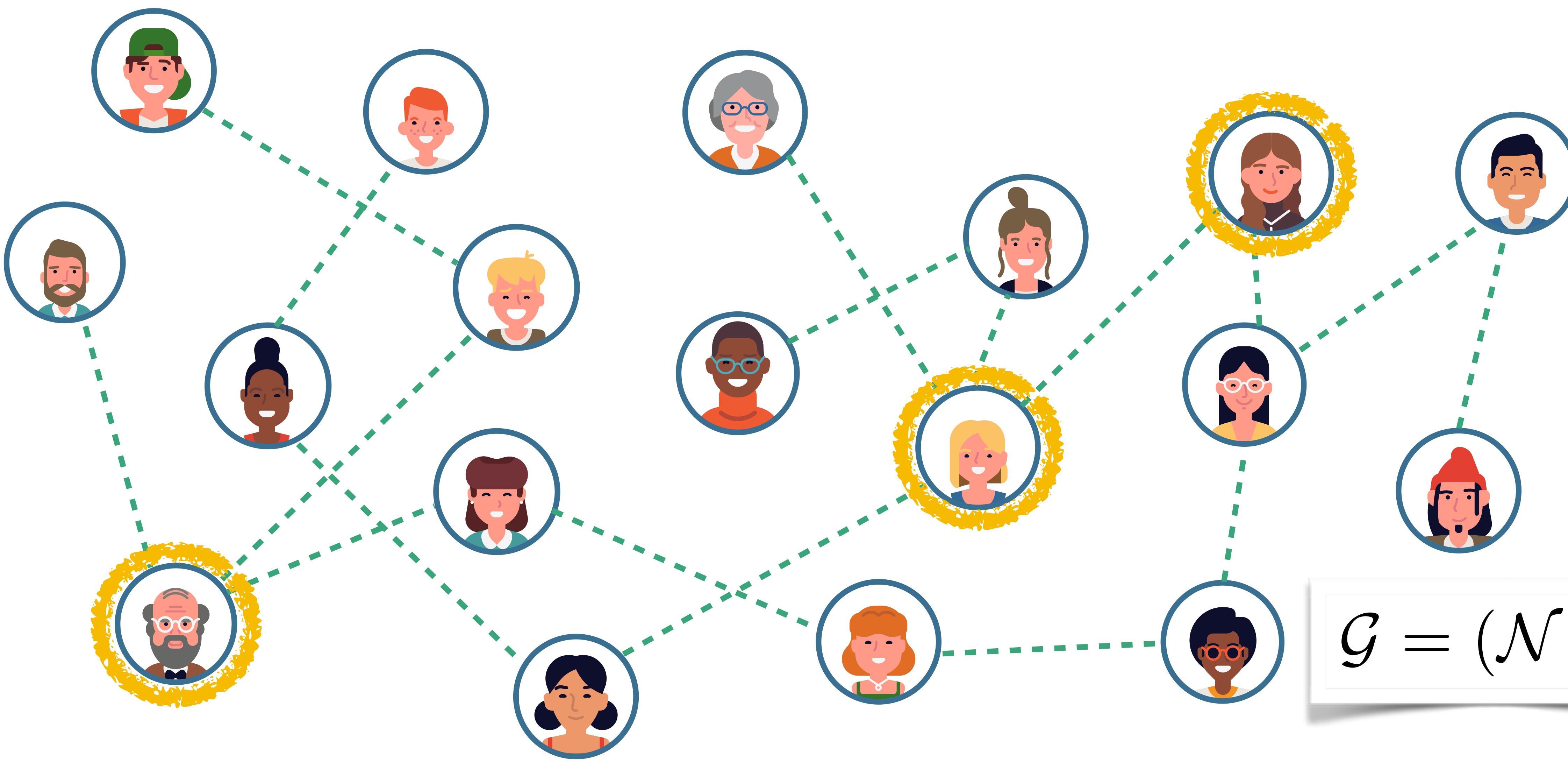
- Combinatorial explosion in number of scenarios

Intervention Model



Known social network: $\mathcal{G} = (\mathcal{N}, \mathcal{E})$

Intervention Model

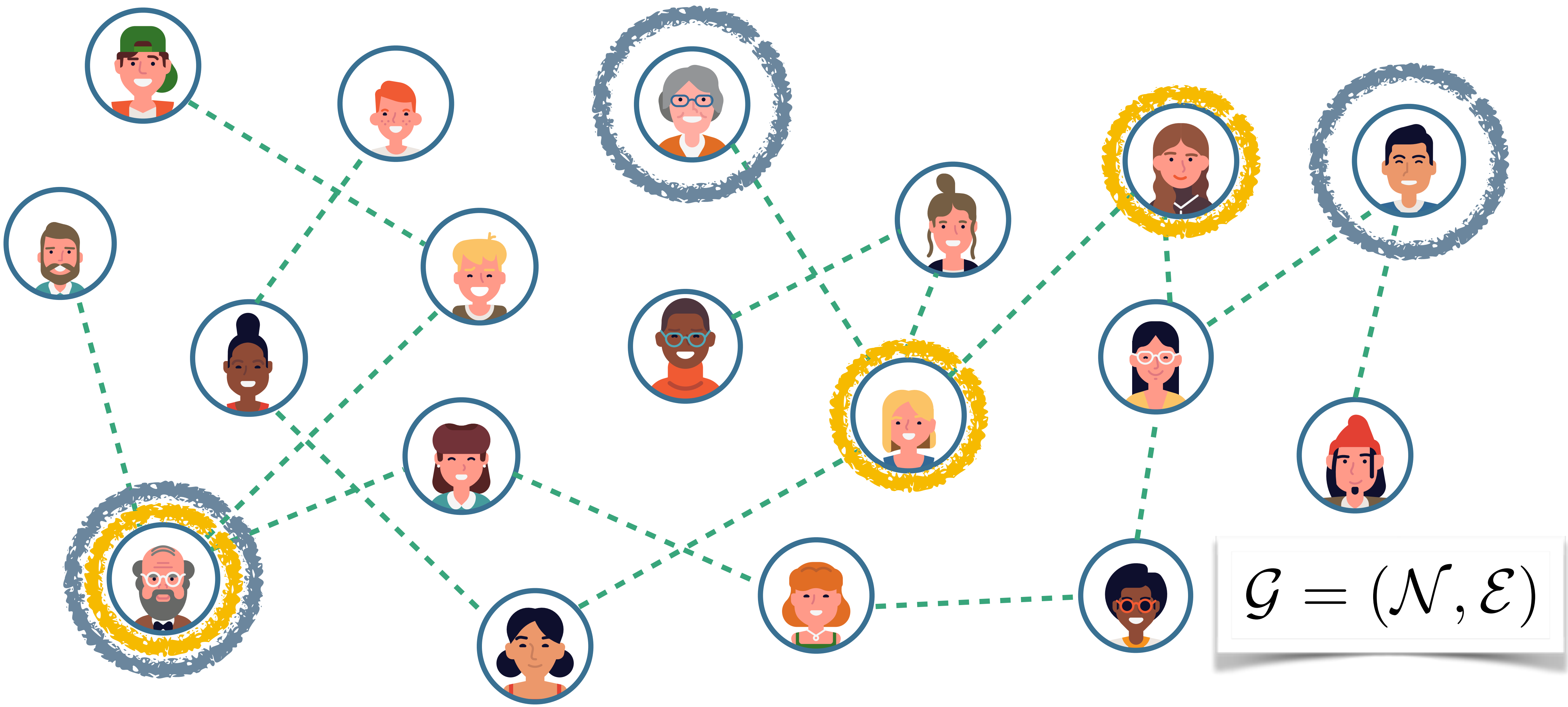


$$\mathcal{G} = (\mathcal{N}, \mathcal{E})$$

Train as a monitor:

$x_n = 1$

Intervention Model



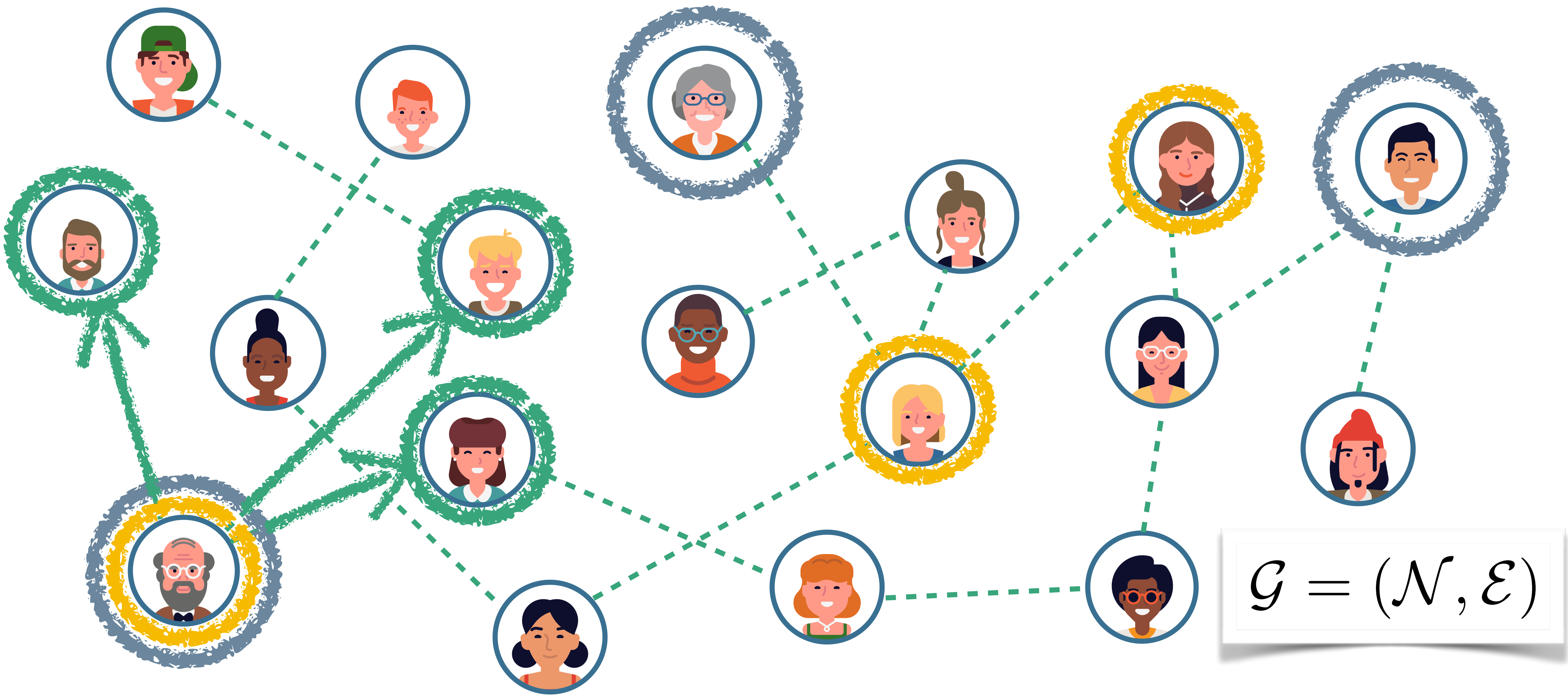
Train as a monitor:

$x_n = 1$

Available:

$\xi_n = 1$

Intervention Model

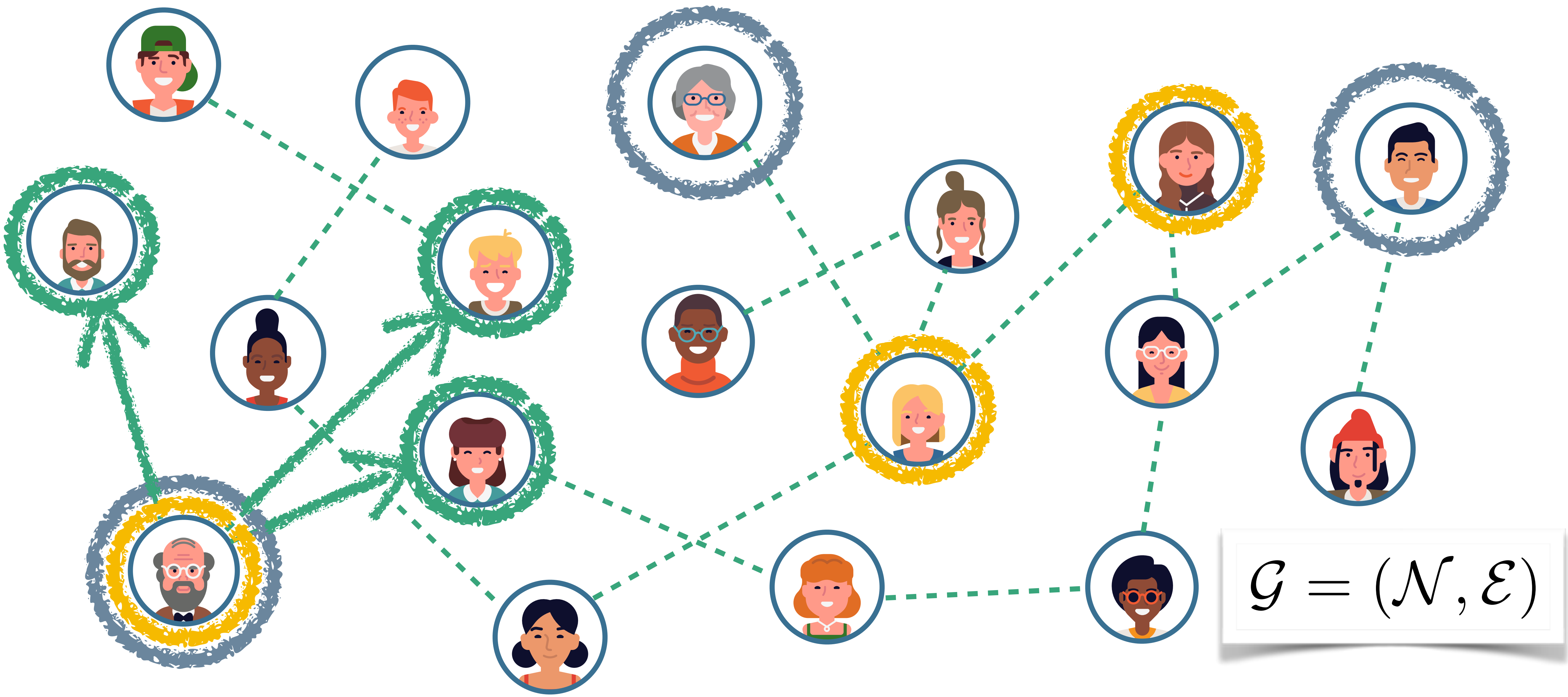


Train as a monitor:
$x_n = 1$

Available:
$\xi_n = 1$

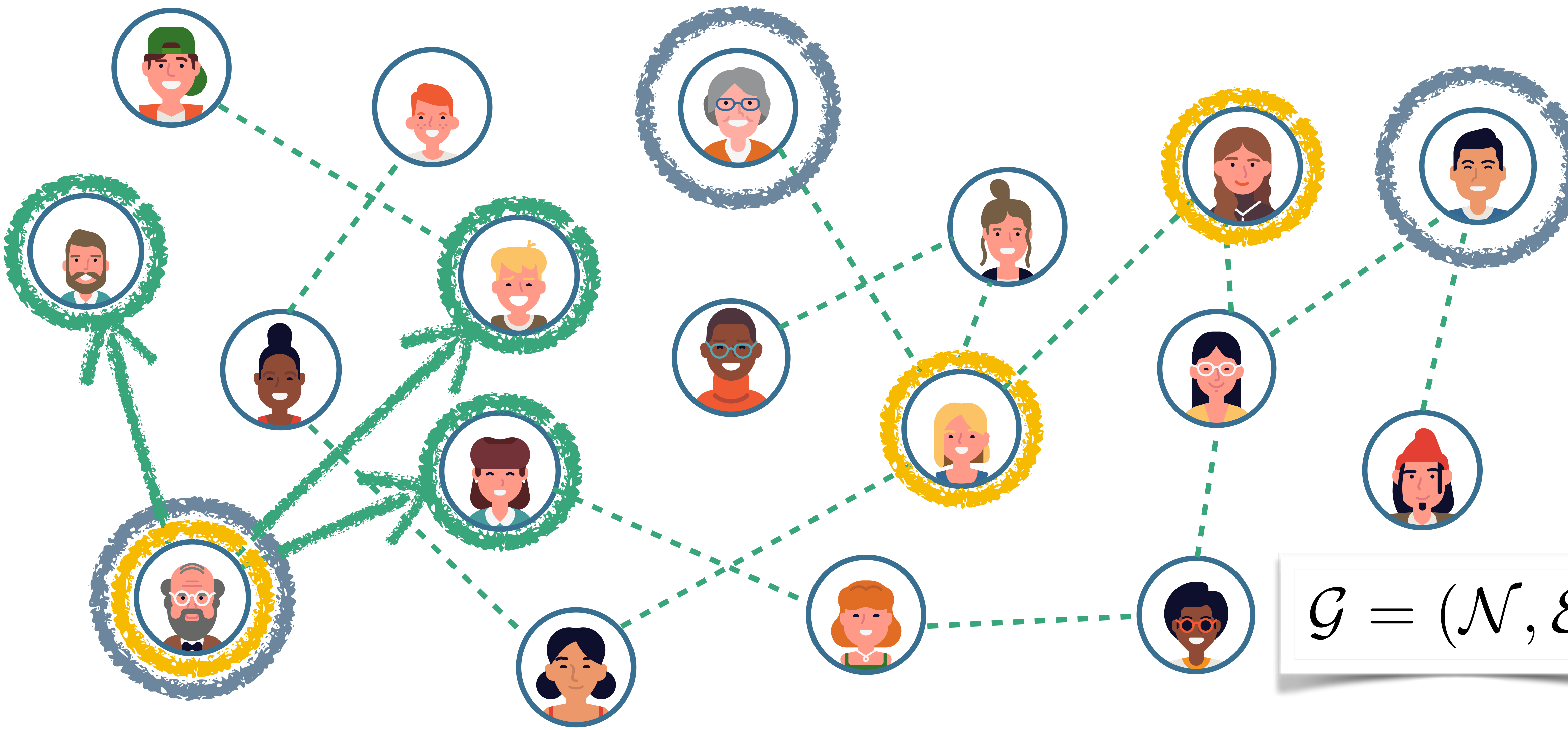
Covered:
$y_n(\boldsymbol{x}, \boldsymbol{\xi}) = 1$

Intervention Model



$$y_n(\boldsymbol{x}, \boldsymbol{\xi}) := \mathbb{I}\left(\sum_{\nu \in \delta(n)} \xi_\nu x_\nu \geq 1\right)$$

Intervention Model



$$\mathcal{G} = (\mathcal{N}, \mathcal{E})$$

$$\boldsymbol{x} \in \mathcal{X} := \{\boldsymbol{x} \in \{0,1\}^N : \mathbf{e}^\top \boldsymbol{x} \leq I\}$$

$$\boldsymbol{\xi} \in \Xi := \{\boldsymbol{\xi} \in \{0,1\}^N : \mathbf{e}^\top (\mathbf{e} - \boldsymbol{\xi}) \leq J\}$$

Intervention Model

Robust Covering

$$\max_{\boldsymbol{x} \in \mathcal{X}} \; \min_{\boldsymbol{\xi} \in \Xi} \; F_{\mathcal{G}}(\boldsymbol{x}, \boldsymbol{\xi}) \;\; \text{where} \;\; F_{\mathcal{G}}(\boldsymbol{x}, \boldsymbol{\xi}) := \sum_{n \in \mathcal{N}} y_n(\boldsymbol{x}, \boldsymbol{\xi})$$

Applying existing algorithm to Social Networks of Youth Experiencing Homelessness?

Existing Greedy Algorithm

Network Name	Size	Percentage Covered by Racial Group				
		White	Black	Hisp.	Mixed	Other
SPY1	95	70	**36**	-	78	89
SPY2	117	77	-	**42**	68	73
SPY3	118	82	-	**33**	81	81
MFP1	165	96	77	69	73	**28**
MFP2	182	**44**	85	70	77	72

Greedy Algorithm: Tzoumas et al. Resilient monotone submodular function maximization

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⇨ Discriminatory Coverage!

Greedy Algorithm: Tzoumas et al. Resilient monotone submodular function maximization

Intervention Model

Robust Covering

$$\max_{\boldsymbol{x} \in \mathcal{X}} \min_{\boldsymbol{\xi} \in \Xi} F_{\mathcal{G}}(\boldsymbol{x}, \boldsymbol{\xi}) \text{ where } F_{\mathcal{G}}(\boldsymbol{x}, \boldsymbol{\xi}) := \sum_{n \in \mathcal{N}} y_n(\boldsymbol{x}, \boldsymbol{\xi})$$

Intervention Model

Robust Covering

$$\max_{\boldsymbol{x} \in \mathcal{X}} \min_{\boldsymbol{\xi} \in \Xi} F_{\mathcal{G}}(\boldsymbol{x}, \boldsymbol{\xi}) \text{ where } F_{\mathcal{G}}(\boldsymbol{x}, \boldsymbol{\xi}) := \sum_{n \in \mathcal{N}} y_n(\boldsymbol{x}, \boldsymbol{\xi})$$

Robust Covering with Fairness Constraints

$$\max_{\boldsymbol{x} \in \mathcal{X}} \min_{\boldsymbol{\xi} \in \Xi} \sum_{c \in \mathcal{C}} F_{\mathcal{G},c}(\boldsymbol{x}, \boldsymbol{\xi})$$

$$\text{s.t.} \quad F_{\mathcal{G},c}(\boldsymbol{x}, \boldsymbol{\xi}) \geq W|\mathcal{N}_c| \quad \forall c \in \mathcal{C}, \ \forall \boldsymbol{\xi} \in \Xi$$

$$\text{where } F_{\mathcal{G},c}(\boldsymbol{x}, \boldsymbol{\xi}) := \sum_{n \in \mathcal{N}_c} y_n(\boldsymbol{x}, \boldsymbol{\xi})$$

Price of Fairness

Price of Group Fairness

$$\mathrm{PoF}(\mathcal{G}, I, J) := 1 - \frac{\mathrm{OPT}^{\mathrm{fair}}(\mathcal{G}, I, J)}{\mathrm{OPT}^{\mathrm{total}}(\mathcal{G}, I, J)}$$

$\mathrm{OPT}^{\mathrm{fair}}(\mathcal{G}, I, J)$: optimal value of fair robust covering

$\mathrm{OPT}^{\mathrm{total}}(\mathcal{G}, I, J)$: optimal value of robust covering

Price of Fairness

Price of Group Fairness

$$\mathrm{PoF}(\mathcal{G}, I, J) := 1 - \frac{\mathrm{OPT}^{\mathrm{fair}}(\mathcal{G}, I, J)}{\mathrm{OPT}^{\mathrm{total}}(\mathcal{G}, I, J)}$$

Deterministic Case:

- Given any $\epsilon > 0$, there exists $\mathcal{G}$ such that:

$$\mathrm{PoF}(\mathcal{G}, I, 0) \geq 1 - \epsilon$$

Price of Fairness

Price of Group Fairness

$$\text{PoF}(\mathcal{G}, I, J) := 1 - \frac{\text{OPT}^{\text{fair}}(\mathcal{G}, I, J)}{\text{OPT}^{\text{total}}(\mathcal{G}, I, J)}$$

Deterministic Case:

- Given any $\epsilon > 0$, there exists $\mathcal{G}$ such that:

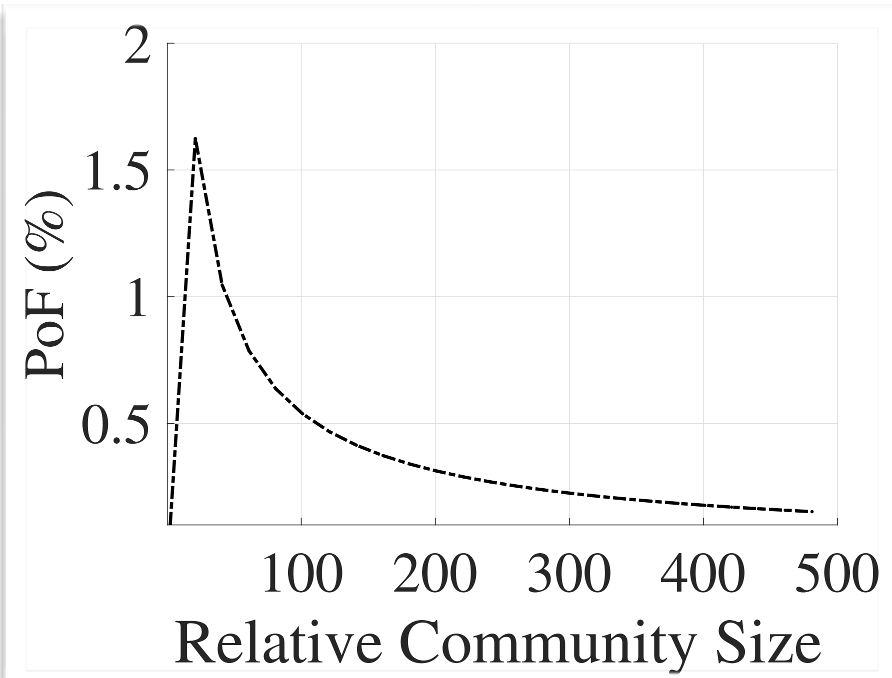
$$\text{PoF}(\mathcal{G}, I, 0) \geq 1 - \epsilon$$

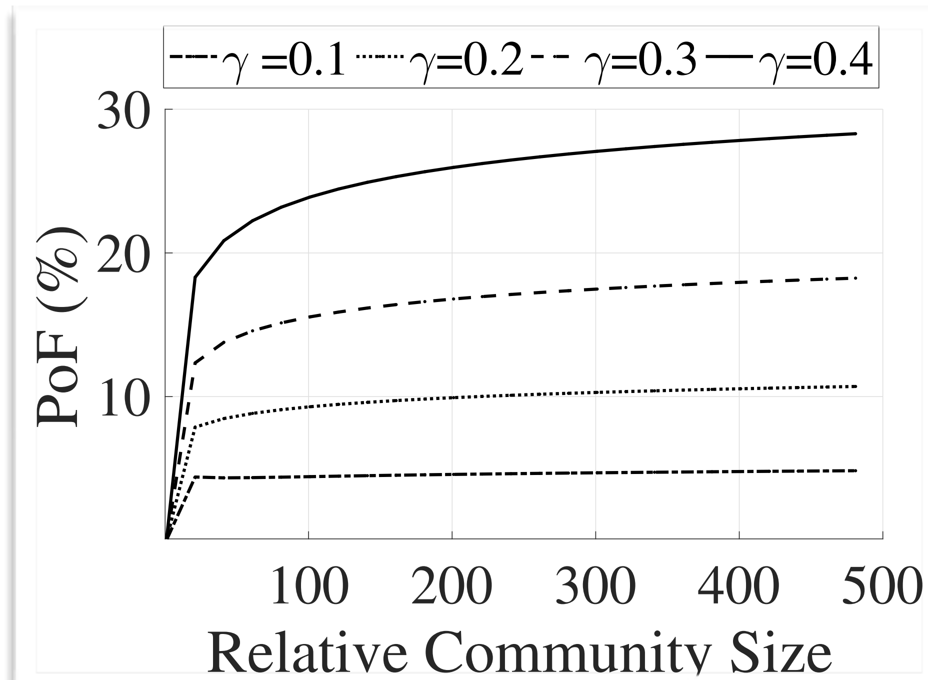
⇒ PoF can be arbitrarily bad!

Expected Price of Fairness

Estimate of Expected Price of Group Fairness

$$\overline{\text{PoF}}(I, J) := 1 - \frac{\mathbb{E}_{\mathcal{G}\sim\text{SBM}}[\text{OPT}^{\text{fair}}(\mathcal{G}, I, J)]}{\mathbb{E}_{\mathcal{G}\sim\text{SBM}}[\text{OPT}^{\text{total}}(\mathcal{G}, I, J)]}$$





We obtain analytical expressions for the expected PoF on SBM networks

Tractable Reformulation

Single-Stage Nonlinear Robust Formulation:

$$\begin{aligned} \max_{\boldsymbol{x} \in \mathcal{X}} \quad & \min_{\boldsymbol{\xi} \in \Xi} \sum_{c \in \mathcal{C}} F_{\mathcal{G},c}(\boldsymbol{x}, \boldsymbol{\xi}) \\ \text{s.t.} \quad & F_{\mathcal{G},c}(\boldsymbol{x}, \boldsymbol{\xi}) \geq W|\mathcal{N}_c| \quad \forall c \in \mathcal{C}, \ \forall \boldsymbol{\xi} \in \Xi \end{aligned}$$

Tractable Reformulation

Single-Stage Nonlinear Robust Formulation:

$$\max_{\boldsymbol{x} \in \mathcal{X}} \quad \min_{\boldsymbol{\xi} \in \Xi} \sum_{c \in \mathcal{C}} F_{\mathcal{G},c}(\boldsymbol{x}, \boldsymbol{\xi})$$

$$\text{s.t.} \quad F_{\mathcal{G},c}(\boldsymbol{x}, \boldsymbol{\xi}) \geq W|\mathcal{N}_c| \quad \forall c \in \mathcal{C}, \ \forall \boldsymbol{\xi} \in \Xi$$

Tractable Reformulation

Two-Stage Linear Robust Formulation:

$$\max_{\boldsymbol{x}\in\mathcal{X}} \min_{\boldsymbol{\xi}\in\Xi} \max_{\boldsymbol{y}\in\mathcal{Y}} \left\{ \sum_{n\in\mathcal{N}} y_n \ : y_n \le \sum_{\nu\in\delta(n)} \xi_\nu x_\nu, \ \forall n \in \mathcal{N} \right\}.$$

where

$$\mathcal{Y} := \{\boldsymbol{y} \in \{0,1\}^N : \sum_{n\in\mathcal{N}_c} y_n \ge W|\mathcal{N}_c| \ \forall c \in \mathcal{C}\}$$

Tractable Reformulation

K-Adaptability Approximation:

$$\begin{aligned} \max \quad & \min_{\boldsymbol{\xi} \in \Xi} \max_{k \in \mathcal{K}} \left\{ \sum_{n \in \mathcal{N}} y_n^k : y_n^k \leq \sum_{\nu \in \delta(n)} \xi_\nu x_\nu, \forall n \in \mathcal{N} \right\} \\ \text{s.t.} \quad & \boldsymbol{x} \in \mathcal{X}, \; \boldsymbol{y}^1, \ldots, \boldsymbol{y}^K \in \mathcal{Y} \end{aligned}$$

Tractable Reformulation

K-Adaptability Approximation:

$$\begin{aligned} \max \quad & \min_{\boldsymbol{\xi} \in \Xi} \max_{k \in \mathcal{K}} \left\{ \sum_{n \in \mathcal{N}} y_n^k : y_n^k \leq \sum_{\nu \in \delta(n)} \xi_\nu x_\nu, \forall n \in \mathcal{N} \right\} \\ \text{s.t.} \quad & \boldsymbol{x} \in \mathcal{X}, \ \boldsymbol{y}^1, \ldots, \boldsymbol{y}^K \in \mathcal{Y} \end{aligned}$$

Equivalent to MILP of polynomial size for any fixed K

Tractable Reformulation

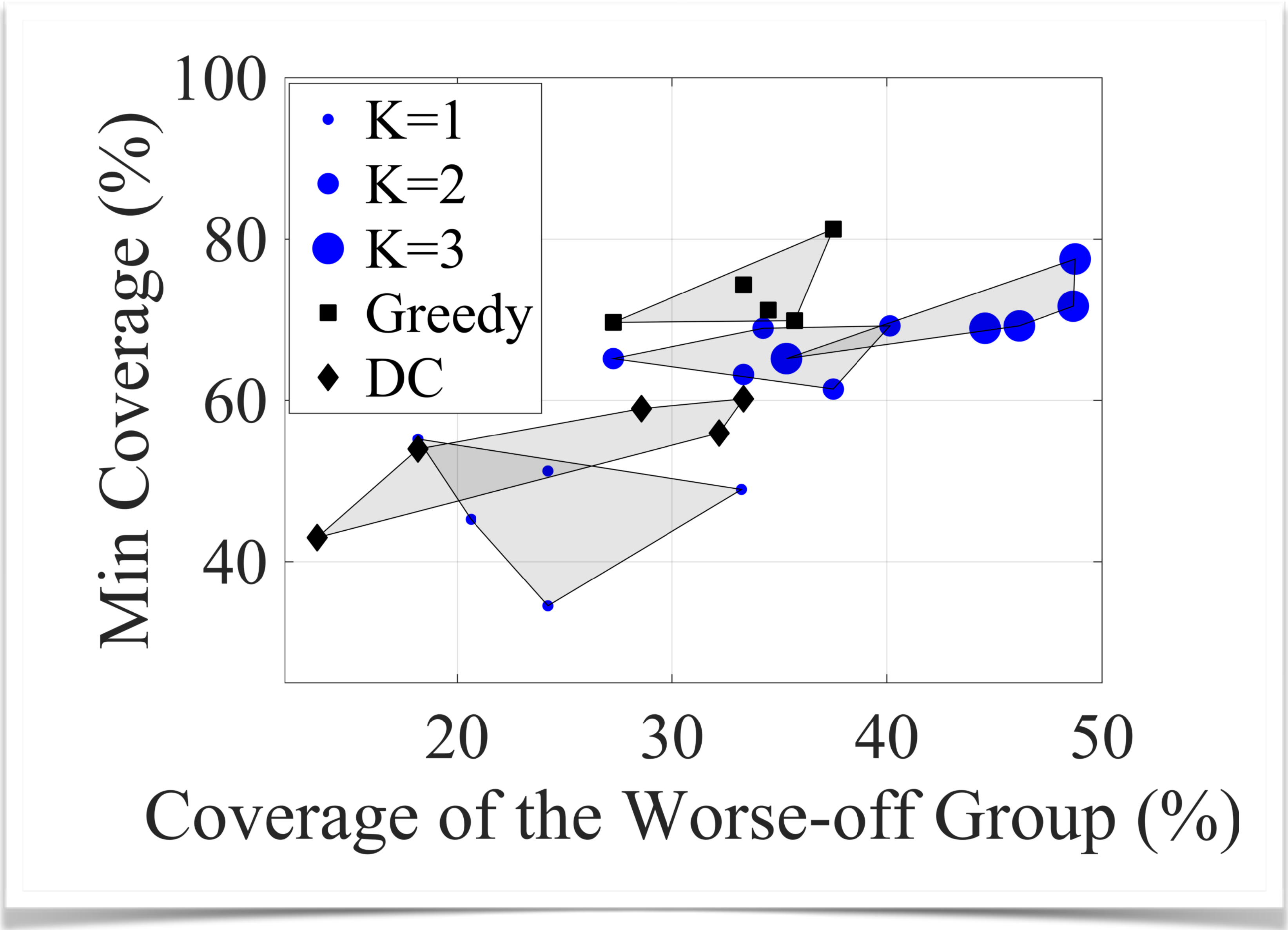
K-Adaptability Approximation:

$$\begin{array}{ll} \max & \min_{\boldsymbol{\xi} \in \Xi} \max_{k \in \mathcal{K}} \left\{ \sum_{n \in \mathcal{N}} y_n^k : y_n^k \leq \sum_{\nu \in \delta(n)} \xi_\nu x_\nu, \forall n \in \mathcal{N} \right\} \\ \text{s.t.} & \boldsymbol{x} \in \mathcal{X}, \ \boldsymbol{y}^1, \ldots, \boldsymbol{y}^K \in \mathcal{Y} \end{array}$$

Equivalent to MILP of polynomial size for any fixed K

Generalizes K-adaptability to discrete uncertainty sets

Numerical Results



Greedy Algorithm: Tzoumas et al. Resilient monotone submodular function maximization

Numerical Results

Network Name	Size	Improvement in Minimum Percentage Covered					
		$J = 0$	$J = 1$	$J = 2$	$J = 3$	$J = 4$	$J = 5$
SPY1	95	15	16	14	10	10	8
SPY2	117	20	14	9	10	8	10
SPY3	118	20	16	16	15	11	10
MFP1	165	17	15	7	11	14	9
MFP2	182	11	12	10	9	12	12
Avg. $I = N/3$		16.6	14.6	11.2	11.0	11.0	9.8
Avg. $I = N/5$		17.2	13.8	14.0	10.0	9.0	6.7
Avg. $I = N/7$		16.4	13.4	11.4	11.4	8.2	6.4

Numerical Results

Network Name	Size	Price of Fairness (%)					
		$J=0$	$J=1$	$J=2$	$J=3$	$J=4$	$J=5$
SPY1	95	1.4	1.0	2.1	4.3	3.3	3.3
SPY2	117	0.0	1.2	3.7	3.3	3.6	3.7
SPY3	118	0.0	3.4	4.8	6.4	3.2	4.0
MFP1	165	0.0	3.1	5.4	2.4	6.3	4.4
MFP2	182	0.0	1.0	1.0	2.2	2.4	3.6
Avg. $I=N/3$		0.28	1.9	3.4	3.7	3.8	3.8
Avg. $I=N/5$		0.2	2.1	3.2	3.2	3.9	3.8
Avg. $I=N/7$		0.2	2.5	3.5	3.2	3.5	4.0

Towards Real World Deployment

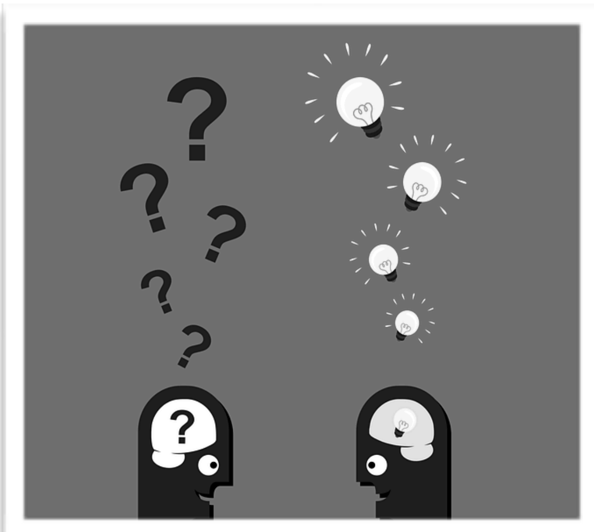
Living to Love Another Day
NICHOLAS H. GUY ACEVEDO MEMORIAL TRUST

USC University of Southern California

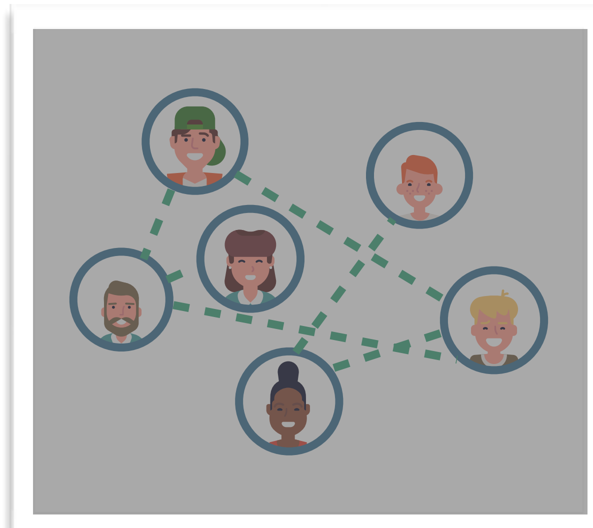
Outline

- ☑ Estimating Wait Times in Resource Allocation Systems
- ☑ Designing Policies for Allocating Scarce Resources
    - ☑ Preference Elicitation
    - ☑ Policy Optimization
- ☑ Optimizing “Gatekeeper Trainings” for Suicide Prevention

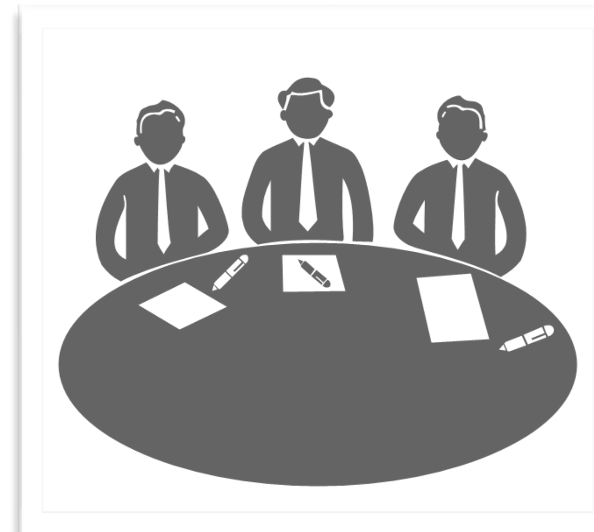
Open Questions



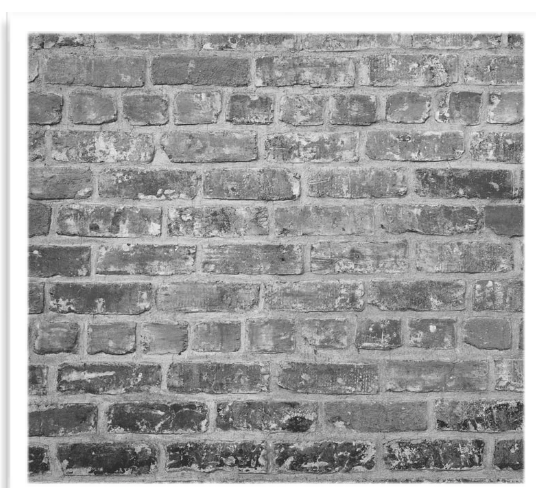
Fair Counterfactual Policy Learning



Unknown Social Networks



Multi-Stakeholder Preferences

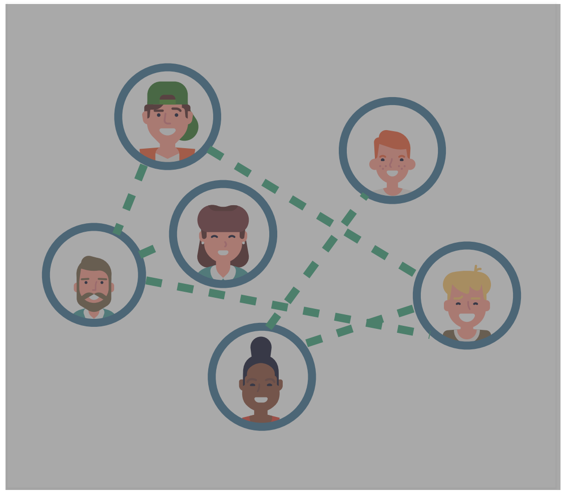


Robust Policies

Conservation

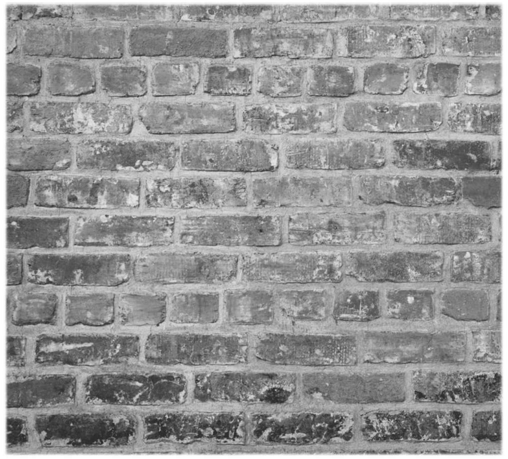
Open Questions

Fair Counterfactual Policy Learning

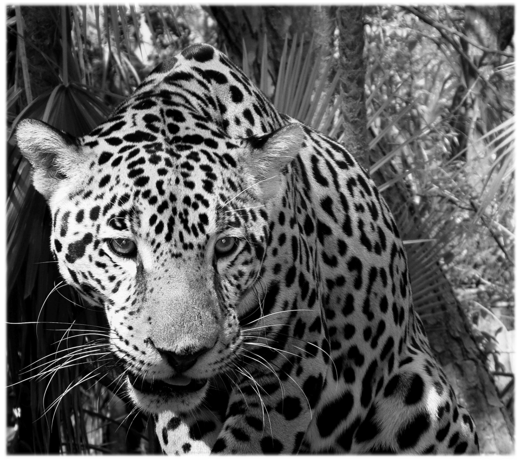


Unknown Social Networks

Multi-Stakeholder Preferences



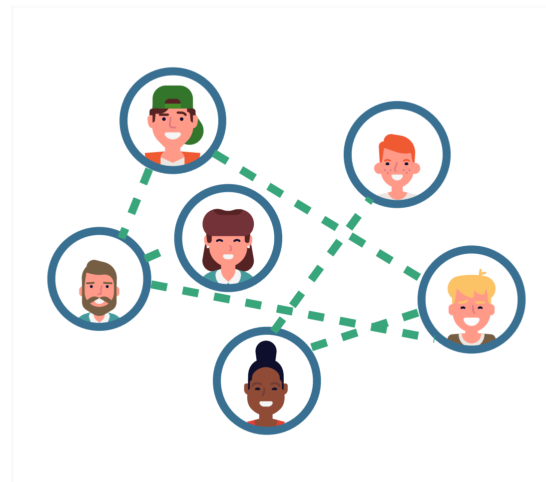
Robust Policies



Conservation

Open Questions

Fair Counterfactual Policy Learning



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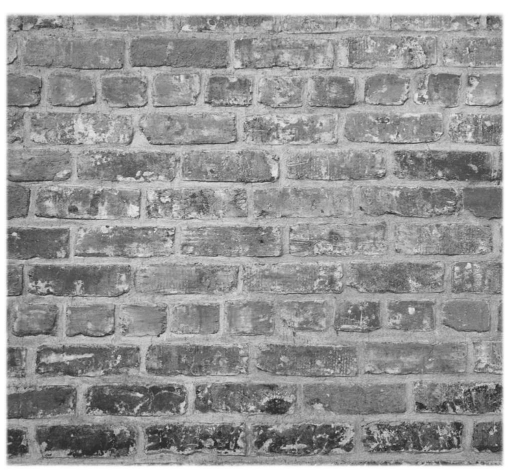
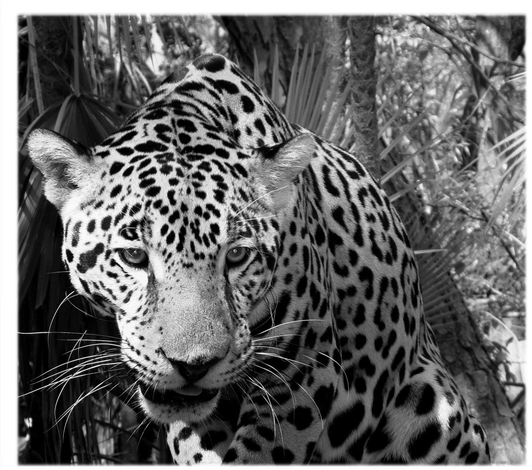


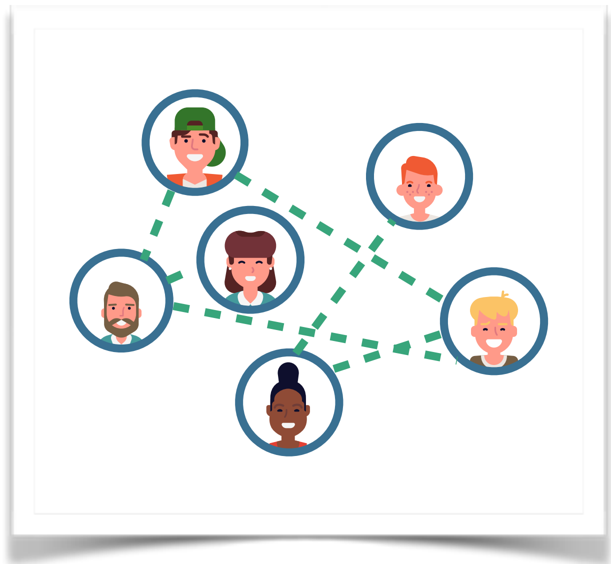
Robust Policies



Conservation

Open Questions

Fair Counterfactual Policy Learning



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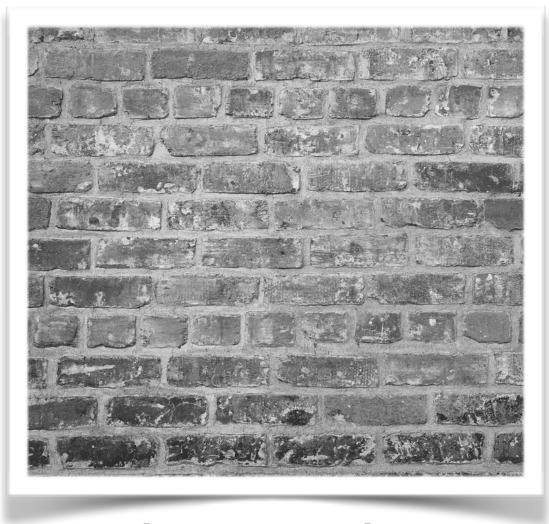

Robust Policies

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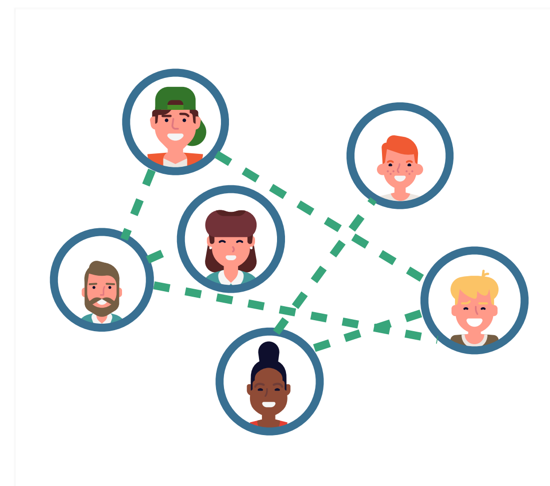
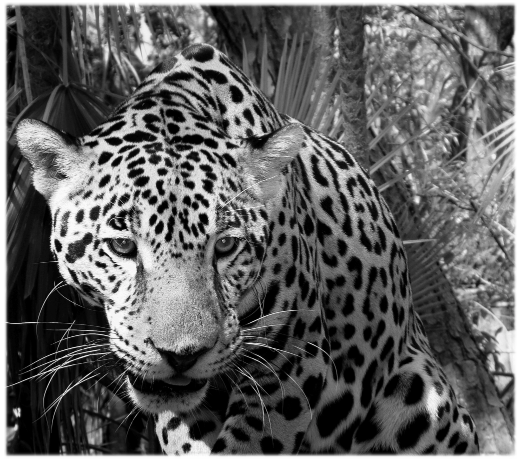
Fair Counterfactual Policy Learning



Unknown Social Networks

Multi-Stakeholder Preferences

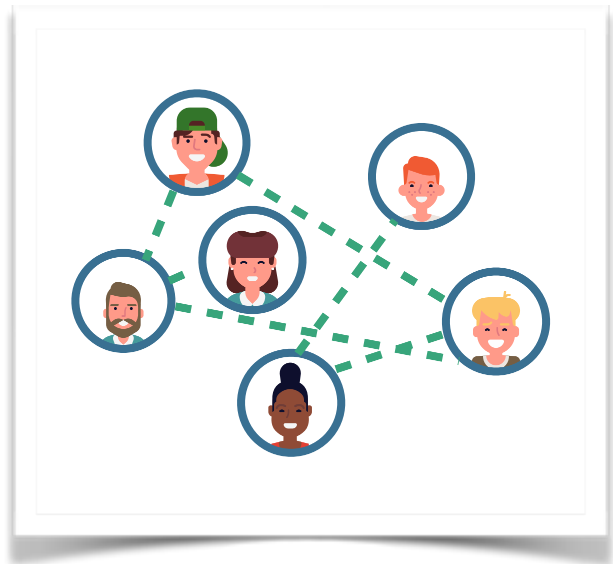
Robust Policies



Conservation

Open Questions

Fair Counterfactual Policy Learning



Unknown Social Networks

Multi-Stakeholder Preferences

Robust Policies

Conservation

Students & CAIS Fellows

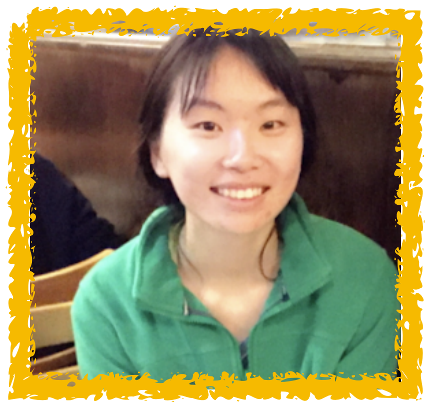
USC PhD Students

Aida

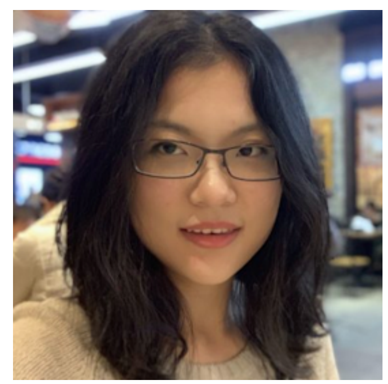
Sina

Ying

Omkar



Han



Qing

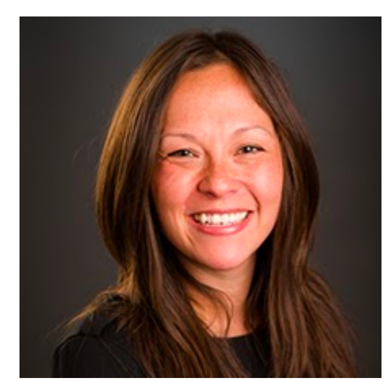
Caroline

Yingxiao

CAIS Summer Fellows

Duncan

Naveena



Jennifer

Hau

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- Zumberge Diversity & Inclusion Grant Program

Presentation Based On:

- Robust multiclass queuing theory for wait time estimation in resource allocation systems, C. Bandi, N. Trichakis and P. Vayanos, *Management Science*, 2018
- Robust optimization with decision-dependent information discovery, P. Vayanos, A. Georghiou, H. Yu, under review at *Management Science*, 2019
- Exploring algorithmic fairness in robust graph covering problems, A. Rahmattalabi, P. Vayanos, A. Fulginiti, E. Rice, B. Wilder, A. Yadav, M. Tambe, *NeurIPS*, 2019
- Robust active preference elicitation, D. McElfresh, Y. Ye, P. Vayanos, J. Dickerson, E. Rice, Working Paper to be submitted to *Management Science*, 2019
- Designing fair, efficient, and interpretable policies for prioritizing homeless youth for housing resources, M. J. Azizi, P. Vayanos, B. Wilder, E. Rice and M. Tambe, *CPAIOR*, 2018

Thank you!

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